Marine Structures 52 (2017) 34–49

Contents lists available at ScienceDirect

Marine Structures

journal homepage: www.elsevier.com/locate/marstruc

The frequency domain estimate of fatigue damage of combined load effects based on the rain-flow counting

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ARTICLE INFO

Article history:
Received 7 September 2016
Received in revised form 1 November 2016
Accepted 28 November 2016

Keywords:
Fatigue damage
Rain-flow counting
Combined processes
Drag force
Inertial force

ABSTRACT

Based on the rain-flow counting technique, a frequency domain method is developed for calculating the fatigue damages caused by the combined loads, which can be linear or nonlinear and Gaussian or non-Gaussian loads. Firstly, the new method is developed for the fatigue damage estimates of the combination of high and low frequency Gaussian loads, by which the ranges of the small rain-flow cycles are obtained as high frequency cycles with considering the reduction effect of low frequency loads on them; the ranges of the large rain-flow cycles are determined by means of Turkstra's rule of load combination. Secondly, the method is extended to calculate the fatigue damages caused by the nonlinear Morison load which is the combination of linear inertial and nonlinear drag forces. The method is benchmarked against the rain-flow damage estimates and compared with the existing ones. The numerical analyses show that the damages estimated by the new method are close to the rain-flow damages.

1. Introduction

It is well accepted that, by the rain-flow counting technique, the structural fatigue damage can be accurately estimated for the varying amplitudes loads [1,2]. But the rain-flow cycles must be extracted based on the known response data. Hence, when structures are subjected to stochastic loading processes, the time domain simulations have to be done so that the rain-flow cycles can be extracted from response processes simulated. Then, based on the rain-flow cycles extracted and Miner' rule, the fatigue lifetime of a structure can be evaluated. As a stochastic process consists of a infinite number of realizations and the time period of the fatigue lifetime estimate is usually very long, the time domain simulation method is time-consuming and not practical one. Besides, the simulation of the non-Gaussian process is still a tough topic.

The alternative way of fatigue damage estimate is the frequency analysis method which is based on the spectral analysis theory of a stochastic process. As the probabilistic distribution of amplitudes of a general non-Gaussian process can not be well defined just based on the spectral density function, the spectral analysis method mainly finds an application in the Gaussian loadings. For Non-Gaussian loadings, it can be used to find the solutions for some special problems based on Gaussian transformations [3–6]. Even for a general wideband Gaussian process, the rain-flow damage estimate [7,8] is also a tough topic because the rain-flow cycles can not well be identified by the spectrum information.

However, when the wideband Gaussian process is the combination of a high frequency (HF) and low frequency (LF) narrowband Gaussian components or a Gaussian bimodal process, the frequency domain analysis method become a powerful

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http://dx.doi.org/10.1016/j.marstruc.2016.11.004
0951-8339/

tool because the rain-flow small and large cycles can be generally determined by the spectrum information of HF and LF loading processes. A lot of works have been done on this issue [9–15].

Obviously, Jiao and Moan [10] appear to be have pioneered the above principle. Based on the principle Benasciutti & Tovo [13] compared different existing methods in the context of rain-flow counting and developed a new one with more applications. Among the works in Refs. [9–13], with the emphasis in finding the simple analytic formulas for the fatigue damage calculation, the amplitudes of the rain-flow large and small cycles were approximated as the sum of amplitudes of high and low frequency loads and those of high frequency loads respectively, which generally cause an conservative estimate of fatigue damages.

Based on the rain-flow counting, Low [14] identified the reduction of the rain-flow small and large cycles, which were named as the effect A and B. Low [14] effectively dealt with the effects A and B by approximating the underlying processes as the harmonic function with the random amplitude. Low's method is accurate and close to the rain-flow estimate. But the derivations of the two coefficients related to reduced effects are very complicated, which make the following numerical calculations rely on the powerful mathematical manipulation of the integrand.

In this paper, based on the author's publication [15], a direct and simple method is developed to consider the effect A and B. Moreover, it has been extended to calculate the fatigue damages caused by non-Gaussian Morison loads.

2. The basic method based on combination of high and low narrow band Gaussian loads

2.1. Identification of rain-flow cycles

The fatigue damage due to the combination of high and low frequency narrow band Gaussian loads is a classical research topic, the relevant practical engineering backgrounds can be referred to [9–15]. With the similar symbols as in these works, the combined process $X(t)$ is expressed as

$$X(t) = X_{HF}(t) + X_{LF}(t) \tag{1}$$

where the $X_{HF}(t)$ and $X_{LF}(t)$ are high and low frequency components.

For such a bimodal stochastic process or even more general stochastic processes, if any realization of them is known, the rain-flow cycles can be extracted from it. Then, based on the S-N curve and the Miner's rule of linear damage accumulation, the fatigue damage corresponding to the realization can be estimated.

However, there are a lot of realizations for a random process and the damages based on different realizations will be different. This means that the fatigue damage is non-deterministic and a random variable. Hence, the mathematic expectation of the damages are much more interested in the fatigue design.

If the n_T rain-flow cycles with ranges $(S_i, i = 1, 2, \cdots n_T)$ have been extracted for one realization of a stochastic process in a given time period T, the damage expectation will be calculated as, see Benasciutti & Tovo [13],

$$D = E\left[\sum_{i=1}^{n_T} \frac{1}{N(S_i)}\right] = \frac{1}{K_r} E\left[\sum_{i=1}^{n_T} S_i^m\right] = \frac{N_T}{K_r} E[S^m] \tag{2}$$

here the $N_T = E[n_T]$ is the mean number of the rain-flow cycles and the $N(S_i)$ is calculated as $N(S_i) = K_r S_i^m$ and $K_r = 2^m K$. The K and m are material parameters of fatigue strength, and

$$E[S^m] = \int_0^\infty s^m f_S(s) \mathrm{d}s \tag{3}$$

The mth moment of the rain-flow range S. The $f_S(s)$ is its density function.

From Eq. (2), it can be seen that the main issue for the estimation of the mean damage is how to get the distribution and the mean number of the rain-flow cycles, especially the former.

If one gets a long enough record of the stress history, The $f_S(s)$ and N_T are can be empirically estimated. However, in the fatigue design stage, there is a lack of data available for estimating them. Moreover, it is much costly and time-consuming. Alternatively, engineers are much more interested in finding them from the spectrum information of a given stochastic process. This is a very challengeable and open topic. In general, there are no general ways to find them.

However, for the bimodal Gaussian processes defined in Eq. (1), the large and small rain-flow cycles can be well identified, the former is mainly formed by the combination of the amplitudes of HF and LF loading processes, the later is mainly contributed by amplitudes of HF loadings, see one typical realization of the bimodal process shown in Fig. 1. For such a bimodal process, the rain-flow ranges $(S_i,\ i = 1, 2, \cdots n_T)$ can be divided two independent subsets: the small cycles $(S_{Si},\ i = 1, 2, \cdots n_{ST})$ and the large ones. $(S_{Li},\ i = 1, 2, \cdots n_{LT})$

Hence, for the bimodal Gaussian process, Eq. (2) can be written as