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A quasi-linear method for frictional model in helical layers of bent flexible risers

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ABSTRACT

This paper deals with the behaviour of two helical layers of bent flexible risers. An analytical model with frictional effects is developed to summarize the mechanical behaviour of helical armour wires. To ensure a unified process of calculation, all formulas are simplified to build up a system of quasi-linear partial differential equations. The numerical solution is determined by finite difference method. Appropriate results are shown in the paper. The geometrical quantities, slips and stresses are matched with other analytical results and found valid.

1. Introduction

In the offshore field, unbonded flexible risers are composite structures and represent a significant part in floating production systems to provide fluid and gas transport. To adapt to the complex marine environment, flexible risers are composed of many reinforced layers which can suffer high deformation in bending behaviour, see Fig. 1. Among these layers, the tensile armour wires are one of the main parts to provide low bending stiffness relative to axial and radial stiffness and the primary cause for stress fatigue failure. Hence, the mechanical analysis of helical armour wire is the emphasis of study in bending research of flexible risers.

The bending behaviour of flexible risers, especially the research focused on tensile armour wires, has been dealt with by many authors throughout the past few decades. Different approaches have been proposed in order to search for an optimized method to predict the structural response. Féret and Bournazel [1] formulated simple equations for calculating the stresses due to that the slip of armour wires follows a geodesic direction, however no evidence has been shown to prove this axisymmetrical loads, evaluating contact pressure and relative slip between layers due to bending. It was assumedchoice. Out and von Morgen [2] considered the Euler's equation to derive the bending stress of a helical wire on a bent cylinder, and calculated the slippage of the wire also with the assumption of geodesic slip.

Witz and Tan [3] developed an analytical model for helical layers of unbonded flexible pipes. The model mainly concluded the overall resultant bending moment before and after slip using energy method. They [3] also described the hysteretic bending moment-curvature relationship. However, some important aspects including the calculation of stresses and slips still remained uncertain. Sævik [4] dealt with a 3D curved beam element by using a computational model where the curvature was prescribed, the constrained element only allowed longitudinal slip. A 2D curved finite element for umbilical analysis was

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Nomenclature

r	radius of a helical layer
R	radius of the bent pipe
θ, φ	angular coordinate
s	wire arclength
α	initial layer angle
$\boldsymbol{T}$	local unit tangential vector of the curve
$\boldsymbol{N}$	local unit normal vector of the surface
$\boldsymbol{B}$	local unit binormal vector
κ_g	geodesic curvature of the wire
κ_n	normal curvature of the wire
τ_g	torsion of the wire
$\boldsymbol{F}$,F	wire sectional force and its component in orthonormal basis
$\boldsymbol{M}$,M	wire sectional moment and its component
$\boldsymbol{p}$,	p wire distributed load and its component
m, m	wire distributed moment and its component
κ	torus curvature $1/R$
ε	relative curvature r/R
E	elastic modulus
G	shear modulus
A	wire cross sectional area
γ	tangential strain of the wire
I_N	wire second N-axial moment of area
I_B	wire second B-axial moment of area
I_P	wire second polar moment of area
Δ_T, Δ_B	wire slips in $\boldsymbol{T}$ and $\boldsymbol{B}$ directions
Δ_θ, Δ_φ	wire slips in angular coordinate directions

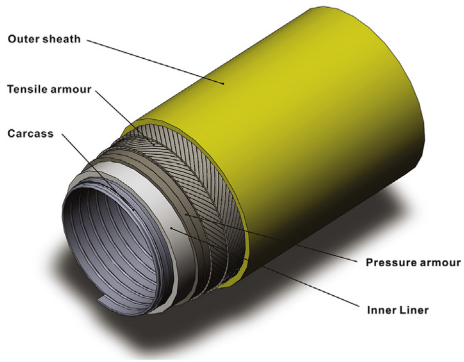

Fig. 1. The structure of flexible pipe.

also reported in Ref. [5]. Subsequently, a model included full back coupling with respect to bending stiffness by means of bi-directional shear interaction was proposed in Ref. [6]. A further development which allowed transverse slip and full 3D was reported in Refs. [7,8]. A research about theoretical and experimental studies of flexible pipes including a bending model and a 3D curved element for umbilical analysis was concluded in Ref. [9]. This work included correlation studies with respect to full scare experimental data in Ref. [4] - [6]. Costello [10] presented a model of wire ropes by applying curved beam equilibrium equations which can be a reliable reference for bending behaviour study.

Ramos and Pesce [11] developed an analytical model for analysing the structure of flexible risers associated with bending, twisting and tension. Using a system of equations including geometrical relations, constitutive equations and equilibrium conditions, all unknowns can be solved based on an assumption of full-slip of the helical layers while subjected to bending. Brack et al. [12] also studied the mechanical behaviour of flexible pipes against the potential failure modes subjected to combined axial compression, bending and torsion by using finite element method. Both theoretical and experimental results were discussed in the paper. Another finite element model was established by Bahtui et al. [13,14]. In the finite-element