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Application of the Modern Taylor Series Method to a multi-torsion chain

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ABSTRACT

In this paper the application of a novel high accuracy numerical integration method is presented for a practical mechanical engineering application. It is based on the direct use of the Taylor series. The main idea is a dynamic automatic order setting, i.e. using as many Taylor series terms for computing as needed to achieve the required accuracy. Previous results have already proved that this numerical solver is both very accurate and fast. In this paper the performance is validated for a real engineering assembly and compared to a Jacobian power series method. The chosen experiment setup is a multi-torsional oscillator chain which reproduces typical dynamic behavior of industrial mechanical engineering problems. Its rotatory dynamics are described by linear differential equations. For the test series the system is operated in a closed-loop configuration. A reference solution of the linear differential equations of the closed-loop system for the output variable is obtained with the mathematical software tool Maple and validated by comparison to measurements from the experiment. The performance of the Modern Taylor Series Method is demonstrated by comparison to standard fixed-step numerical integration methods from the software tool Matlab/Simulink and to the Jacobian power series approximation. Furthermore, the improvement in numerical accuracy as well as stability is illustrated and CPU-times for the different methods are given.

1. Introduction

In many industry applications simulation has become a powerful tool for engineers in order to predict real system behavior or validate closed-loop controller performance without conducting cost-intensive test cycles on appropriate test facilities. Often the specific problem setup requires the application of accurate algorithms for fixed step solutions. Simulation speed, accuracy as well as stability of the performing algorithms are critical in such applications (see e.g. [1]).

Today's most common mathematical simulation software packages (e.g. Matlab/Simulink) provide various types of numerical integration methods [2]. These methods differ primarily in the way the solution at the next time step is calculated, knowing the time derivative at the current time step. Variable-step solvers are able to adapt the interval step size dynamically during simulation, based on the current rate of change of the solution. Fixed-step solvers predefine a fixed step size because they need to calculate the simulation output deterministically for each time step. Increasing the step size, fixed-step algorithms typically become unstable at a certain step size limit [3–5]. More information about numerical methods and stability can be found in [6]. Finding a fixed-step numerical integration method which is accurate and fast therefore increases the quality of such algorithms drastically.

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A very promising approach for such problems is the Modern Taylor Series Method (MTSM) [7] – a special parallel system which has been developed at the Brno University of Technology. This parallel system can be used in a special hardware unit for the acceleration of numerical integration. The main component of the parallel system is a numerical integrator carrying out numerical integration based on the Taylor series approximation. A description of this system can be found in [8].

Some articles that are focused on the MTSM were published recently. In [9] a new modern numerical method based on the Taylor series method is described, and it is shown how to evaluate the high accuracy and speed of the corresponding computations. Article [10] deals with the simulation system TKSL that has been created for the numerical solution of the first-order differential equations using the MTSM.

There are several papers that focus on computer implementations of the Taylor series method in different context – a variable order and *variable step* formulation of the Taylor method for the numerical solution of ODEs (see, for instance, [11]). Another more detailed description of a variable step size version of the Taylor series can be found in [12]. This paper describes a software implementation of the method as well. The stability domain for several Taylor methods is presented in [5]. An implicit version of the Taylor series method for power system simulations is presented in [13] but is computationally much more involved. In [14] a comparison of Taylor series methods to matrix exponential methods for nonlinear ordinary differential equations (ODEs) with input delays is presented.

In order to compare the performance of the MTSM to standard fixed-step solvers and a Jacobian Power Series Method (JPSM) a laboratory experimental model of a multi-torsional oscillator is utilized. This system represents a typical example for mechatronic applications because it can serve as a reference model for drive train components of internal combustion engines. It can be modeled by linear ODEs using theoretical modeling techniques and can be simulated using common numerical simulation tools, e.g. Matlab/Simulink. Measurements for all relevant variables can be recorded with minimum effort. Furthermore, the closed-loop control of this system demonstrates a characteristic use case within automatic control applications. This paper is based on [15], but substantially extends those results by the treatment of the JPSM, an analysis on stability of the algorithms, a revised section on modeling, and a quantitative comparison of CPU time needed.

The paper is structured as follows: Section 2 contains a detailed description of the MTSM and is followed by Section 3, which presents the JPSM for comparison. In Section 4 a stability analysis of the MTSM and JPSM methods is presented. Section 5 is devoted to the mathematical modeling and the experimental setup. Section 6 provides tests and comparisons of the presented numerical integration methods, and Section 7 concludes the paper.

2. Modern Taylor Series Method

The Taylor series method is one of the earliest analytic-numeric algorithms for the approximate solution of initial value problems for ODEs. Even though this method is not much preferred in literature, experimental calculations have shown and theoretical analyses have verified that the accuracy and stability of the Taylor series method exceed those of currently used algorithms [7,10].

2.1. Fundamental principle

Consider the ordinary differential Eq. (1) with initial condition:

$$\dot{y} = f(t, y), \quad y(t_0) = y_0. \tag{1}$$

The best-known and most accurate method of calculating a new value of a numerical solution for an ODE is to construct the Taylor series approximation in the form (2)

$$y_{n+1} = y_n + h \cdot f(t_n, y_n) + \frac{h^2}{2!} \cdot f^{[1]}(t_n, y_n) + \cdots + \frac{h^p}{p!} \cdot f^{[p-1]}(t_n, y_n), \tag{2}$$

where $h \in \mathbb{R}$ is the integration time step and $p \in \mathbb{N}$ is the order of the finite approximation.

The main idea behind the MTSM is an automatic integration method order setting, i.e. using as many Taylor series terms as necessary to achieve the required accuracy. MTSM adapts the order automatically, i.e. the values of the terms (3) are computed for increasing integer values of p until a termination criterion is met. This is assumed to hold if the last three terms of the Taylor series approximation are smaller than a predefined threshold ε:

$$\frac{h^j}{j!} \cdot f^{[j-1]}(t_n, y_n) < \varepsilon, \quad j = p+1, \ p+2, \ p+3. \tag{3}$$

Note that a suitable ε in (3) may be chosen using a standard Lagrange error bound [16]; the inclusion of three consecutive terms results from experience and aims for a more robust termination criterion. The main problem connected with using the Taylor series in the form of (2) is the need to generate higher derivatives $f^{[1]}, \ f^{[2]}, \ \ldots$ If it is possible, however, to obtain these terms, the achievable accuracy of MTSM is extreme (it is in fact only limited by the type of the arithmetic unit used). A drawback of this method is that $f(t, y)$ has to belong to a special class. Fortunately, this class is obviously large enough to contain the functions that appear in many applications. This is typical, in particular, of the solution of technical initial problems [17,18].