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Nonlinear Lamb wave-mixing technique for micro-crack detection in plates

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ABSTRACT

In engineering practice, failures due to fatigue cracks in metallic structures have always been difficult to predict. In our study, nonlinear Lamb wave-mixing was applied in the detection of micro-cracks in plates. The analysis of the nonlinear interaction of these waves with cracks of various lengths and widths was performed using finite-element simulations. The simulation results showed that the sideband at the sum frequency provide a sensitive means for micro-crack detection. Moreover, the sideband amplitudes show a monotonic increase with micro-crack length, but a decrease with micro-crack width. Experiments using Lamb wave-mixing were conducted on plates with fatigue cracks. The experimental results show that a proposed acoustic nonlinearity parameter related to the sideband at the sum frequency is sensitive to micro-cracks in plate, and is well correlated with the damage degree.

1. Introduction

Metallic plates are widely used in mechanical, civil, and aerospace applications. Failure in these metallic structures under time-varying loads is often attributed to cracks developed over time through fatigue. Therefore the detection of micro-cracks at an early stage of fracture is important to avoid catastrophic failures of engineering components and structures. Lamb waves has been widely used in the field of nondestructive testing of plates as they can transfer energy over long distances and detect internal defects by interrogating the entire thickness of the structure [1,2]. The traditional Lamb wave technique is based on linear theory and normally relies on measuring some particular parameters, such as acoustic velocity, attenuation, transmission and reflection coefficients, to determine the elastic properties of a material or to detect defects. The presence of defects changes the phase and/or amplitude of the output signal, but the frequency of the input and output signals is the same. However, the conventional ultrasonic technique (including Lamb wave) is sensitive to gross defects or opens cracks, where there is an effective barrier to transmission, whereas it is much less sensitive to evenly distributed micro-cracks or degradation [3,4].

An alternative technique to overcome this limitation is the non-linear ultrasonic technique. The principal difference between linear and non-linear ultrasonic techniques is that for the latter the existence and characteristics of defects are often related to an acoustic signal for which the frequency differs from that of the input signal. According to the principle of detection, nonlinear ultrasonic methods can be divided into harmonic [5,6], resonance [7], vibro-acoustic modulation [8], and mixing wave method [9]. The experimental research demonstrated that the nonlinear ultrasonic techniques are robust to factors such as complicated geometry or moderate environmental variations, such as wind and temperature, therefore they have unique advantage in field applications [10,11].

Attempts have been made recently to apply the nonlinear behavior of Lamb waves for nondestructive evaluation (NDE) and material characterization [12,13]. The research is mainly focused on the generation of second harmonics in the Lamb wave response, which has been theoretically and experimentally investigated for micro-structural damages, such as plasticity, material degradation, and fatigue damages in metal. For example, Deng [14] investigated complicated problems of second-harmonic generation of Lamb waves using a second-order perturbation approximation and a modal analysis approach. Deng [15] and Pruell et al. [16] investigated the feasibility of using nonlinear effects of Lamb waves for assessing accumulated fatigue damage in solid plates. Pruell et al. [17] developed an experimental procedure for characterizing fatigue damage in metallic plates using nonlinear Lamb waves. Xu et al. [18] theoretically and experimentally investigated the harmonic generation effect of non-linear Lamb waves in aluminum plates using time-frequency analysis. Li et al. [19] applied nonlinear Lamb waves for the detection of material degradation caused during thermal cycles. The major difficulty for the application of harmonic generation method is to isolate the causes of nonlinearity. As there are inevitable nonlinear distortions in the transmitting/receiving system, such as amplifiers, transducers, and coupling media, it is difficult in practice for the harmonic generation technique to determine if the measured nonlinearity is because of

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damage or the testing system.

An alternative method to overcome these limitations is the wave mixing technique, which is based on the fact that material nonlinearities cause an interaction between two intersecting ultrasonic waves [20,21]. Under certain circumstances, this can lead to the generation of a third wave for which frequency and wave-vector equal the sums of those of the incident waves. The generated resonance wave is related to the nonlinearity of materials/structures. Therefore, by measuring this wave, the nonlinearity might be obtained. Compared to the generation of second harmonics, the advantages of wave mixing technique includes: it is less sensitive to nonlinearities in the measurement system, and allows for great flexibility in selecting wave modes, frequencies, and propagation directions [21]. Although there have been many studies of the bulk waves mixing using for NDE and material characterization, little research has been conducted on the Lamb wave mixing for damage detection [20,21]. Lee et al. [22] conducted an exploratory study on using the nonlinear Lamb wave-mixing technique for damage detection in plates. This technique was shown to have potential to measure the nonlinearity of the micro-structures.

In this paper, we report our investigation both theoretical and experimental of this technique in nondestructive testing of fatigue cracks in plates. First, the fundamental theory of nonlinear Lamb wave mixing was introduced. Second, an analysis of the nonlinear interaction of Lamb waves with cracks of various lengths and widths was performed using finite-element (FE) simulation. Finally, experiments were conducted on Lamb wave mixing in fatigued plates for micro-crack detection.

2. Propagation of a nonlinear Lamb wave

2.1. Propagation of linear Lamb waves

Lamb waves are defined as elastic waves of plane strain propagating in a traction-free, homogeneous and isotropic plate. The propagation of such waves is governed by the Navier equation

$$(\lambda_0 + \mu_0)u_{j,ji} + \mu_0 u_{i,jj} + \rho b_i = \rho \ddot{u}_i, \tag{1}$$

where λ_0 and μ_0 are the Lamé constants, ρ is the density, b_i the force per unit mass, u the displacement, and i, j are coordinate indices. By applying the traction-free boundary condition, the dispersion equation can be derived,

$$\frac{\tan qh/2}{\tan ph/2} = -\left(\frac{4k^2pq}{(q^2-k^2)^2}\right)^{\pm 1}, \tag{2}$$

where exponent ± 1 specifies the symmetric/antisymmetric mode, h is the plate thickness, k the wave number, and p and q are defined as

$$p = \sqrt{k_l^2 - k^2}, \quad q = \sqrt{k_t^2 - k^2}, \tag{3}$$

where k_l and k_t are the wavenumber amplitudes for the longitudinal and transversal waves, respectively. As indicated by (2), the propagation of Lamb waves has dispersive, multi-mode characteristics. Fig. 1 shows the phase and group velocity dispersion curves for a 1.7 mm-thick steel plate.

The multi-mode and dispersive nature of Lamb waves complicate interpreting the received signals. Therefore, single-mode waves are desirable in NDE applications. In this paper, the fundamental symmetric mode (S0) was chosen as mode for nonlinear Lame wave mixing measurement. The reasons include: S0 is almost non-dispersive, which makes the interpretation of signals easier; Moreover, it is the fastest mode, therefore it is the first wave-packet to arrive at the receiver; In addition, the displacement of the S0 mode is almost uniform through the thickness of the plate, thus its sensitivity to defects is independent of their through-thickness location. The excitation frequencies in Lamb wave mixing measurements were determined by considering of various factors, such as the dispersion characteristic of S0 mode, frequency responses of transmitter and receiver, and resonance characteristic of test samples. According to the dispersion characteristic of S0 mode, wave mixing experiments should be conducted in lower frequency range due to the lower dispersion. The detailed process of excitation frequencies determination were discussed in Section 4.1. In this paper, we report on the simulated and experimental studies of nonlinear Lamb wave mixing using S0 mode at low frequencies of 450 kHz and 600 kHz.

2.2. Nonlinear response of Lamb wave mixing

Physically, the phenomenon of wave mixing response is related to nonlinearity in the elastic behavior of the material, which indicates that the relationship between stress σ and strain ε is nonlinear. This nonlinear relationship can be described by the nonlinear Hooke law:

$$\sigma = E\varepsilon(1 + \beta\varepsilon + \cdots), \tag{4}$$

where E is Young's modulus and β the second-order nonlinear elastic coefficient.

Assuming that the nonlinearity in the plate is small, the solution to (1) for time-harmonic waves can be calculated by perturbation analysis. The solution to (1) is assumed to take the form

$$u(x, t) = u^{(0)} + \beta u^{(1)}, \tag{5}$$

where $u^{(0)}$ and $u^{(1)}$ represent the general solution and second-order perturbation solution, respectively. Generally, the perturbation solution is assumed to be proportional to the propagation distance,

$$u^{(1)} = xf(\tau), \tag{6}$$

where $\tau = t - x/c$ and $f(\tau)$ is the undetermined function.

If the excitation contains two sinusoidal components of different frequencies for the S0 mode, the general solution can be expressed as

$$u^{(0)}(x, t) = A_1\cos(f_1 t - k_1 x) + A_2\cos(f_2 t - k_2 x), \tag{7}$$

where pairs k_1, k_2, A_1, A_2, and f_1, f_2 are respectively the wavenumbers, amplitudes, and frequencies of the two sinusoidal components. The unknown function $f(\tau)$ can be determined by substituting (6) and (7) into (5) and subsequently into (1). This gives

$$f(\tau) = -\frac{A_1^2k_1^2}{8}\cos(2f_1\tau) - \frac{A_2^2k_2^2}{8}\cos(2f_2\tau) + \frac{A_1A_2k_1k_2}{4}[cos(f_1 - f_2)\tau - cos(f_1 + f_2)\tau] \tag{8}$$

Therefore, the perturbation solution (5) becomes

$$\begin{aligned} u(x, t) &= u^{(0)} + \beta u^{(1)} \\ &= A_1\cos(f_1 t - k_1 x) + A_2 cos(f_2 t - k_2 x) \\ &+ x\beta\Big\{-\tfrac{A_1^2k_1^2}{8}\cos(2f_1 t - 2k_1 x) - \tfrac{A_2^2k_2^2}{8}\cos(2f_2 t - 2k_2 x) \\ &+ \tfrac{A_1A_2k_1k_2}{4}\{cos[(f_1 - f_2)t - (k_1 - k_2)x] - cos[(f_1 + f_2)t - (k_1 + k_2)x]\}\Big\}. \end{aligned} \tag{9}$$

This expression gives the time–space response of the nonlinear system under a mixed-frequency excitation. Because of the nonlinearity in the stress-strain relationship, the two sinusoidal components interact with each other, from which appears the sidebands at the sum and difference frequencies of the excitations.

Considering this nonlinear solution of the wave equation for two sinusoidal excitations, the acoustic nonlinearity parameter β can be written in the form

$$\beta = \frac{4A_{f_1+f_2}}{A_1A_2k_1k_2x}, \tag{10}$$

where $A_{f_1+f_2}$ is the amplitude of the sideband at the sum frequency of the excitations. As shown in the above equation, the proposed acoustic nonlinearity parameter is related to the propagation distance, the