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Bifurcation analysis of the simplified models of boiling water reactor and identification of global stability boundary

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HIGHLIGHTS

- Non-linear stability analysis of nuclear reactor is carried out.
- Global and local stability boundaries are drawn in the parameter space.
- Globally stable, bi-stable, and unstable regions have been demarcated.
- The identification of the regions is verified by numerical simulations.

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ABSTRACT

Nonlinear stability study of the neutron coupled thermal hydraulics instability has been carried out by several researchers for boiling water reactors (BWRs). The focus of these studies has been to identify subcritical and supercritical Hopf bifurcations. Supercritical Hopf bifurcation are soft or safe due to the fact that stable limit cycles arise in linearly unstable region; linear and global stability boundaries are same for this bifurcation. It is well known that the subcritical bifurcations can be considered as hard or dangerous due to the fact that unstable limit cycles (nonlinear phenomena) exist in the (linearly) stable region. The linear stability leads to a stable equilibrium in such regions, only for infinitesimally small perturbations. However, finite perturbations lead to instability due to the presence of unstable limit cycles. Therefore, it is evident that the linear stability analysis is not sufficient to understand the exact stability characteristics of BWRs. However, the effect of these bifurcations on the stability boundaries has been rarely discussed. In the present work, the identification of global stability boundary is demonstrated using simplified models. Here, five different models with different thermal hydraulics feedback have been investigated. In comparison to the earlier works, current models also include the impact of adding the rate of change in temperature on void reactivity as well as effect of void reactivity on rate of change of temperature. Using the bifurcation analysis of these models the globally stable region in the parameter space has been identified. The globally stable region has only stable solutions and does not have even unstable limit cycles. Hence, the system is stable irrespective of the size of the perturbation in these regions.

1. Introduction

The nonlinear dynamical system of nuclear reactors is very complex and can be modelled with coupled partial differential equations. In the current work, the phenomenological study of this complex system has been carried out by reduced order models consisting of coupled ordinary differential equations. These coupled ordinary differential equations of nuclear reactors may reveal many complex nonlinear phenomena which have not been investigated yet. The linear stability analysis of nuclear reactors has been of paramount research interest in last few decades, but analysis was limited to Hopf bifurcation and limit cycles. Therefore, this work focuses on detailed understanding of subcritical (hard and dangerous) and supercritical Hopf (soft and safe) bifurcations; and especially their consequences like bi-stability in the system and bifurcation of limit cycles. Here, a nonlinear stability boundary has been discussed which differentiates globally (nonlinear) stable region and locally (linear) stable region.

The neutron coupled thermal-hydraulic instabilities are one of the most important instability, which occur in nuclear reactors due to the reactivity feedback mechanism. Coefficients of void

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and fuel temperature of reactivity are one of the important reasons for the appearance of nonlinearity in the system. The nonlinearities in the BWRs are also present due to energy and momentum conservation equations; however nonlinearities due to these factors have not been incorporated here.

Wang and Kondo (1992) has demonstrated effect of different modelling approximations on local stability and bifurcations for BWRs. The centre manifold method is used for the analysis of fifth order phenomenological model of BWR presented by March-Leuba et al. (1986a,b). However, the study is limited to local stability boundary and limit cycles.

The limit cycles are manifestation of Hopf bifurcation which are closed trajectories that attract or repel its nearby trajectories. The first investigation of these limit cycles in the nuclear reactor dynamical system has been carried out in March-Leuba et al., 1986a,b. This model has five coupled nonlinear ordinary differential equations but covers the essential features of BWRs. The neutronics equations are point reactor kinetics equations which assume that neutron density is same throughout reactor core. The second order reactivity equation and the differential equation for rate of change of temperature of the fuel rod completes the mathematical model. The above model has been discussed in several subsequent works such as Tsuji et al. (1993), Rizwan-Uddin (2006), Bindra and Rizwan-uddin (2014). Rizwan-Uddin (2006) implemented semi-analytical bifurcation theory to predict subcritical and supercritical Hopf bifurcations and to draw linear stability boundaries for different parametric regions. This work has a brief discussion on turning point bifurcation of limit cycles. However, its implication for the stability of the system and especially identification of the regions in the parametric space where these bifurcations occur is not discussed.

Bindra and Rizwan-uddin (2014) has investigated the impact of effect of rate of change of temperature on void reactivity as well as effect of void reactivity on rate of change of temperature by making approximations to the March-Leuba model. However, this analysis is limited to linear stability boundary.

Lange et al. (2014) discussed stability from a nonlinear point of view with a representative reduced order model (ROM-RAM) for BWR. This work presents the fact that the decay ratio stability analysis misses the bifurcation phenomena which occur in the BWRs dynamical systems and bifurcation phenomena are nonlinear behaviour whereas the decay ratio predicts only linear stability characteristics.

The above mentioned works were focussed on linear stability boundaries and limit cycles associated with the Hopf bifurcation. However, no effort has been made to identify the globally stable region, where an only linear phenomenon exists without any limit cycles. The existence of subcritical Hopf bifurcation serves as a motivation to search for nonlinear (global) stability boundary in the system. Therefore, this work investigates linear and nonlinear stability boundaries for the five different reduced order models of a BWR. These models are characterized by gradual increase of complexity in the mathematical representation of the thermal hydraulics. The first model shows linear unstable behaviour only. However, as the damping term in reactivity is included due to subcritical Hopf, unstable limit cycles emerge. The unstable limit cycle captures the dynamics of the system as fuel and coolant heat transfer process is included. The stable limit cycles and supercritical Hopf are manifested for this mathematical representation of the system. The effect of rate of change of temperature on the void reactivity is included in the fourth model. The effect of void reactivity on the rate of change of temperature is included in the fifth model. These effects have been investigated from linear as well as nonlinear point of view.

The nonlinearity in the systems shifts the exact (nonlinear or global) stability boundary from the linear stability boundary. This nonlinearity contributes to an unstable phenomenon in a linear stable regime, here in this work unstable limit cycle is that unstable phenomenon. The boundary at which unstable limit cycles gain stability via limit point bifurcation of limit cycles (LPCs) by the emergence of stable limit cycles over unstable limit cycles; is the nonlinear stability boundary. The nonlinear stability boundary is important for the design of any physical nonlinear dynamical system as it identifies values of parameters where the system is globally stable.

This analysis of nonlinear stability boundary has been done widely in other fields such as chemical oscillators, biological systems, power systems, ecology, tourism, economics, and neurology, etc. The bi-stability for subcritical Hopf bifurcation has been mentioned as corridor stability in the work of Kind (1999) for economic interpretations. Subramanian et al. (2010) did work on non-linear stability boundary for horizontal Rijke tube. The bi-stability pattern and sub-criticality in car following model for transportation were studied by Orosz and Stépán (2006).

The emphasis of this paper is the study of complex dynamics of the system from the design point of view. The emergence of stable limit cycles from the nonlinear stability boundary in the linearly stable region along with an unstable limit cycle leads to bi-stability in the system. A precise identification of the globally stable, bi-stable and unstable region of the system has been made for the desirable parametric region.

Pandey and Singh (2016) adopted March-Leuba model and mathematically explained this limit point of bifurcation of limit cycles and period doubling bifurcation of limit cycles on the different parametric regimes. In the present work, five different models which include several important physical phenomena are studied with focus on nonlinear (global) stability. Moreover, the earlier work identified a few bifurcations mathematically, however, their impact on the design of the BWRs was not discussed. Here, the impact on the stability analysis of various model approximations is also demonstrated by using a hierarchy of BWR models.

Here, the implementation of bifurcation analysis is carried out via numerical continuation technique. This technique is path following algorithms which take initial solutions and can trace out the trajectory corresponding to the equilibrium solution as a parameter of the system varies. The MatCont bifurcation package, a MATLAB® based platform is used to achieve the goal in this work (Dhooge et al., 2003). The bifurcation diagrams on the parameter plane have been drawn on this platform whereas the phase space plots have been drawn using Ode45 solver of MATLAB®.

2. Bifurcation analysis

The sudden qualitative change in the behaviour of a dynamical system with the change of values of one or more parameters is referred as bifurcation phenomena. This phenomenon cannot be predicted by linear stability analysis and the drastic change in the dynamical system because of this phenomenon can lead to instability. These instabilities can be predicted by bifurcation analysis of the system, and this study improves the understanding of system design and makes it safer as this analysis predict the accurate stability of the system. This analysis helps to predict marginal distance from the linear stability boundary for the stable operational parameters of the system. The detailed mathematical study of bifurcation theory can be read in Kuznetsov (2004). However, some basic background is presented here for the general readers.

2.1. Hopf bifurcation

Hopf bifurcation is a well-known local and co-dimension one bifurcation that occurs in the dynamical system of nuclear reactors