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One-dimensional two-fluid model for wavy flow beyond the Kelvin–Helmholtz instability: Limit cycles and chaos

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A B S T R A C T

A 1D TFM numerical simulation of near horizontal stratified two-phase flow is performed where the TFM, including surface tension and viscous stresses, is simplified to a two-equation model using the fixed-flux approximation. As the angle of inclination of the channel increases so does the driving body force, so the flow becomes KH unstable, and waves grow and develop nonlinearities. It is shown that these waves grow until they reach a limit cycle due to viscous dissipation at wave fronts. Upon further inclination of the channel, chaos is observed. The appearance of chaos in a 1D TFM implies a nonlinear process that transfers energy intermittently from long wavelengths where energy is produced to short wavelengths where energy is dissipated by viscosity, so that an averaged energy equilibrium in frequency space is attained. This is comparable to the well-known turbulent stability mechanism of the multi-dimensional Navier–Stokes equations, i.e., chaos implies Lyapunov stability, but in this case it is strictly a two-phase phenomenon.

1. Introduction

It is well known that the one-dimensional two-fluid model (1D TFM) may be rendered well-posed once appropriate short wavelength physics is incorporated. For example the surface tension force makes the TFM well-posed for horizontal stratified flows beyond the Kelvin–Helmholtz (KH) instability (Ramshaw and Trapp, 1978). This is a proper physical solution to the linear stability problem, but the finite exponential wave growth remains. However, the TFM is inherently non-linear, and little is known about its non-linear stability. The purpose of this paper is to investigate the Lyapunov stability of a 1D TFM beyond the KH criterion.

The state-of-the-art of the 1D TFM stability analysis remains more or less where it was when the present generation of US TFM nuclear reactor safety codes were written in the early seventies, that is, within the realm of linear stability theory. Later advances in the field of non-linear dynamics and chaos have not transcended yet into the understanding of the stability of the TFM.

In the first place Whitham (1974) elaborated a set on non-linear solutions to the two-equation shallow-water theory (SWT) consisting of shocks and expansion waves and identified the kinematic SWT instability. But SWT differs from TFM in one important aspect: it does not include the dynamic KH instability. Beyond that, Kreiss and Yström (2002) (KY) analysed a two-equation model that is dynamically similar to the TFM beyond the KH instability. They obtained shocks and expansion waves similar to SWT and observed that the viscous force limits the growth of the waves. Furthermore, Fullmer et al. (2014a) showed that the KY equations are chaotic.

Recently Lopez de Bertodano et al. (2013) derived the two-equation fixed-flux model from the TFM that reduces exactly to SWT for flow conditions below the KH instability, thus rendering the TFM amenable to Whitham's analyses. The fixed-flux model is based on the fixed flux assumption, which allows local instabilities like SWT and KH, but precludes global instabilities like flow excursion and density waves. In this paper the fixed-flux model is applied to perform a stability assessment of Thorpe's experiment (Thorpe 1969) beyond the initial wave growth period, including linear analysis and nonlinear simulations, resulting in limit cycles and chaos.

2. Fixed-flux two-equation model

The incompressible fixed-flux TFM of Lopez de Bertodano et al. (2013) obtained from the full TFM of Fullmer et al. (2014b) and validated with the experiment of Thorpe (1969) is given by:

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Nomenclature

c	wave speed ($\mathrm{m\,s^{-1}}$)
C	coefficient of void gradient term in momentum equation ($\mathrm{m^2\,s^{-2}}$)
$C(r)$	number of points of a trajectory contained in hypersphere of radius r
f	friction factor
g	acceleration due to gravity ($\mathrm{m\,s^{-2}}$)
H	channel height (m)
j	volumetric flux ($\mathrm{m\,s^{-1}}$)
k	wave number ($\mathrm{m^{-1}}$)
L	test section length (m)
r	radius
r_ρ	density ratio
u	velocity ($\mathrm{m\,s^{-1}}$)
Greek letters	
α	volume fraction
λ	wavelength (m)
μ	dynamic viscosity (Pa s)
ν	kinematic viscosity ($\mathrm{m^2\,s^{-1}}$)
ρ	density ($\mathrm{kg\,m^{-3}}$)
σ	surface tension ($\mathrm{N\,m^{-1}}$)
θ	angle of channel inclination (rad)
ω	angular frequency ($\mathrm{s^{-1}}$)
Subscripts	
1	heavier phase
2	lighter phase
i	interfacial
ρ	density ratio
Acronyms	
KH	Kelvin–Helmholtz
SWT	shallow water theory
TFM	two-fluid model
1D	one dimensional

$$\frac{\partial\alpha_1}{\partial t}+u_1\frac{\partial\alpha_1}{\partial x}+\alpha_1\frac{\partial u_1}{\partial x}=0, \tag{1}$$

$$\frac{\partial u_1}{\partial t}+\frac{1-\alpha_1}{1-\alpha_1+\alpha_1 r_\rho}\left(u_1+r_\rho\frac{\alpha_1}{1-\alpha_1}(2u_2-u_1)\right)\frac{\partial u_1}{\partial x} -\frac{1-\alpha_1}{1-\alpha_1+\alpha_1 r_\rho}C\frac{\partial\alpha_1}{\partial x} =\frac{1-\alpha_1}{1-\alpha_1+\alpha_1 r_\rho}\left(\frac{\sigma H}{\rho_1}\frac{\partial^3\alpha_1}{\partial x^3}+F_{visc}+F\right), \tag{2}$$

where,

$$C=\left[r_\rho\frac{(u_2-u_1)^2}{1-\alpha_1}-(1-r_\rho r)g_y H\right] \tag{3}$$

is the void convection coefficient and the algebraic drag terms are grouped as

$$F=g_x-\frac{1}{\alpha_1 H}\frac{f_1}{2}|u_1|u_1+\frac{1}{\alpha_2 H}\frac{f_2}{2}r_\rho|u_2|u_2 +\left(\frac{1}{\alpha_1 H}+\frac{1}{\alpha_2 H}\right)\frac{f_i}{2}r_\rho|u_2-u_1|(u_2-u_1). \tag{4}$$

Finally, it can be shown that the combined viscous force, assuming the viscosity is the same for both phases, is:

$$F_{visc}=\nu\left[\left(\frac{1}{\alpha_1}+\frac{r_\rho}{\alpha_2}\right)\frac{\partial}{\partial x}\alpha_1\frac{\partial u_1}{\partial x}+\frac{r_\rho}{\alpha_2}\frac{\partial}{\partial x}\frac{u_1}{\alpha_2}\frac{\partial\alpha_1}{\partial x}\right]. \tag{5}$$

Two more equations are needed for closure. The first is the void fraction restriction

$$\alpha_1+\alpha_2=1. \tag{6}$$

Secondly we consider the total flux to express the velocity of one component in terms of the other. By combining the time derivative of Eq. (6) with the sum of the phasic continuity equations one gets:

$$\frac{\partial}{\partial t}(\alpha_1+\alpha_2)+\frac{\partial}{\partial x}(\alpha_1 u_1+\alpha_2 u_2)=\frac{\partial j}{\partial x}=0 \tag{7}$$

where j is the total volume flux.

Eq. (7) shows that, provided that the phase densities are constant, j is spatially uniform, i.e., $j(x,t)=j(t)$. For the present case of Thorpe (1969), stagnated flux restriction applies, i.e., $j(t)=0$. This key assumption greatly simplifies the TFM equations without removing the local material instabilities.

If C is negative and surface tension is neglected, the two-equation fixed-flux model in the limit $r_\rho\to 0$ becomes the well-known 1D SWT equations (Whitham 1974; Wallis 1969). Furthermore $C=0$ leads to the long wavelength Kelvin–Helmholtz criterion, e.g., see Eq. (2–147) of Ishii and Hibiki (2006),

$$(u_2-u_1)^2>\frac{1-r_\rho}{r_\rho}(1-\alpha_1)g_y H \tag{8}$$

If C is positive the equations represent the Kelvin–Helmholtz unstable regime which is the case of the TFM, beyond the scope of SWT, under study. We are now in a position to define the types of waves and instabilities that will be analysed.

The dynamic wave speed, derived later, is given by $c=\sqrt{-\alpha_1 C}$ and the corresponding instability condition is $C>0$, associated with the dynamic KH instability. On grounds of the analogy between the TFM and SWT, it may be stated that the linear and non-linear behaviour of the dynamically stable TFM (i.e., $C<0$) may be understood in terms of the many well-known results derived in SWT. If $C=0$ and $F=0$ the system becomes the water faucet model of Ransom (1984) which is of practical interest to the verification of the TFM for nuclear reactor safety codes. The case $C>0$ corresponds to the dynamically unstable incompressible TFM, and it is of unique interest to two-phase flow analysis in general and reactor safety codes in particular, because it is ill-posed when surface tension is not included. However, the nonlinear behaviour of the well-posed case has not been explored beyond the pioneering mathematical analyses of Kreiss and Yström (2002) and Keyfitz et al. (2004).

2.1. Viscous term

Additional constitutive equations are required for the closure of the wall and interfacial shear terms and the effective viscosities. For the present calculations the values are $f_1=f_2=0.005$, and $f_i=0.014$. More importantly, the effective viscosity needs to be specified. Alas, a complete model for the turbulent viscosity is not presently available. In its stead, a rough, order-of-magnitude model is proposed here, which hopefully suffices for the numerical simulations. Since the densities of the Thorpe experiment (1969) are quite close, a first-order approximation is to neglect the

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