Global dissipativity in the mean square of stochastic Cohen-Grossberg neural networks with time delays

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Abstract: In this paper, the problem of global dissipativity in the mean square is discussed for stochastic Cohen-Grossberg neural networks with time delays. By constructing general Lyapunov functions, combining with Itô's formula, several sufficient conditions for the global dissipativity in the mean square are derived. Moreover, we give out the estimations of globally attractive sets. Finally, one example is given to show the effectiveness of the proposed criteria.

Key Words: Stochastic Cohen-Grossberg neural networks, dissipativity in the mean square, Itô's formula, attractive set

1 Introduction

The Cohen-Grossberg neural networks(CGNNs) model, which was first investigated by Cohen and Grossberg in 1983 [1]. A lot of researchers have attracted considerable attention on analysis of CGNNs due to its potential applications, and a great number of important and interesting results have been obtained on the analysis of CGNNs, including stability analysis, state estimation [2, 3, 4, 5, 6]. In the past few years, the dynamical behaviors of stochastic neural networks have emerged as a new subject of research mainly, in real systems, the synaptic transmission is a noisy process brought on by random fluctuations [6] from the release of neurotransmitters and other probabilistic causes [7, 8], and it has been realized that a neural network could be stabilized or destabilized by certain stochastic inputs [9, 10]. In particular, the stability criteria for stochastic neural networks become an attractive research problem of prime importance [11, 12].

But, in many applications, it is possible there are multiple equilibriums with some being unstable [13]. When the neural network is used as associative memory storage or for pattern recognition, the existence of many equilibrium is also necessary. In these applications, the neural networks are no longer globally stable, and more appropriate notions of stability are needed to deal with multistable system. In addition, the past decade researchers have witnessed the rapid development of dissipative theory [14, 15] in system and control areas, because the dissipative theory gives a framework for the design [16] and analysis of control systems using an input-output description based on energy-related considerations, and the dissipative theory serves as a powerful tool in characterizing important system behaviors, such as stability and passivity, and has close connections with passivity theorem, bounded [17, 18, 19]. Until now, much effort has been focused on the stability of stochastic Cohen-Grossberg neural networks(SCGNNs) [2, 3, 4]. Unfortunately, so far, few authors research the dissipative analysis problem for the SCGNNs with delays. Therefore, the main purpose of this paper deal with the dissipativity analysis problem for SCGNNs with time delays.

There are three different definitions about stochastic neural networks: 1) ultimately bounded in probability, 2) almost surely ultimately bounded, 3) pth moment ultimately bounded. Here, we just consider the pth moment ultimately bounded, especially the ultimately bounded in the mean square(when $p = 2$). The notion "global dissipativity in the mean square" for the SCGNNs is introduced instead of the general dissipativity notion [20]. Based on the Itô's formula and some inequality analytic techniques, sufficient conditions are established to ensure the dissipativity. Moreover, the globally attractive set in the mean square and the positive invariant in the mean square are given explicitly.

2 Preliminaries

The SCGNNs model with time delays is described by the following equation group:

$$\begin{cases} dx_i(t) = \{\alpha_i(x_i(t))[-h_i(x_i(t)) + \sum\limits_{j=1}^{n} a_{ij} f_j(x_j(t)) \\ \qquad\qquad + \sum\limits_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_j)) + u_i]\}dt \\ \qquad\qquad + \sum\limits_{j=1}^{n} \sigma_{ij}(t, x_i(t), x_i(t-\tau_i))d\omega_j(t), \\ x_i(t) = \varphi_i(t), -\tau \le t \le 0, \end{cases} \tag{1}$$

where $i = 1, 2, \cdots n, n \ge 2$ is the number of neurons in network (1), $x_i(t)$ denotes the state variable associated with the neuron, and $\alpha_i(x_i(t))$ is an appropriately behaved function. Function α_i is continuous. The connection matrix $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$, activation function $f_j(\cdot)$

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shows us how the neurons respond to each other, a_{ij}, b_{ij} are connection weights from neuron i to neuron j; u_i is a constant external input; τ_j denotes the discrete time delays, $\varphi_i(t) \in C([-\tau, 0]; \mathbb{R})$ is the initial condition for neural networks (1).

Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of $n \times m$ matrices. Let $X > 0$ (respectively ,$X \geq 0$) means that X is symmetric positive definite (respectively positive semi-definite) matrix. $C^b_{\mathscr{F}}([-\tau, 0] : \mathbb{R}^n)$ is the family of all $\mathscr{F}$-measurable bounded $C([-\tau, 0] : \mathbb{R}^n)$-valued random variables, and for $\tau > 0$, $C([-\tau, 0] : \mathbb{R}^n)$ denotes the family of continuous function φ from $[-\tau, 0]$ to $\mathbb{R}^n$, with the norm $||\varphi|| = sup_{-\tau\leq s\leq 0}|\varphi(s)|$, where $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$. For the convenience of discussion, let

$$x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^{\mathrm{T}},$$
$$f(x) = (f_1(x_1), f_2(x_2), \cdots, f_n(x_n))^{\mathrm{T}},$$
$$U = (u_1, u_2, \cdots, u_n)^{\mathrm{T}},$$
$$\sigma(t, x(t), x(t-\tau)) = (\sigma_{ij}(t, x_i(t), x_i(t-\tau_i)))_{n\times n}.$$

We rewrite Eq.(1)

$$\begin{cases} dx(t) = \{\alpha(x(t))[-h(x(t)) + Af(x(t)) + Bf(x(t \\ \qquad -\tau)) + U]\}dt + \sigma(t, x(t), x(t-\tau))d\omega(t), \\ x(t) = \varphi(t), -\tau \leq t \leq 0, \end{cases} \tag{2}$$

Next, we give definition of three types of activation functions. The first class of activation functions include sigmoid functions. It consists of all bounded continuous function. Formally, we define the set

$\mathscr{B} := \{g_i(\cdot)|g_i \in C(R, R), \exists k_i > 0, |g_i(x_i)| \leq k_i, \forall x_i \in R, i = 1, 2, \cdots, n\}$,

The second class of activation functions is of Lurie type, which may be unbounded and includes Lipschitz function. It is defined by

$\mathscr{H} := \{g_i(\cdot)|g_i \in C(R, R), \exists l_i > 0, |g_i(x_i)| \leq l_i|x_i|, \forall x_i \in R, i = 1, 2, \cdots, n\}$. Let $L = \max\{l_i\}$.

The third class of activation functions is a general type, which may be neither bounded nor monotonous or differenial. There exist constant l_i^-, and l_i^+ such that

$\mathscr{G} := \{f_i(\cdot)|l_i^- \leq \frac{f_i(x)-f_i(y)}{x-y} \leq l_i^+,\ \forall x,\ y \in R,\ x \neq y,\ i = 1, \cdots, n\}$.

Remark 1. Set $\mathscr{G}$ is less conversation than $\mathscr{H}$, since the constants l_i^-, and l_i^+ are allowed to be positive, negative or zero, that is to say, the activation function is assumed to be neither monotonic, nor differentiable, nor bounded. In addition, when $f(0) = 0$, and $l_i = max\{|l_i^+|, |l_i^-|\}$, we have $|f_i(x)| \leq l_i|x|$, so the set $\mathscr{H} \subseteq \mathscr{G}$.

Throughout this paper, we suppose the following assumptions hold.

Assumption (H1): For each $i \in \{1, 2, \cdots, n\}$, the amplification function $\alpha_i(\cdot)$ is positive, bounded, and satisfies $0 < \underline{\alpha_i} \leq \alpha_i(x_i(t)) \leq \overline{\alpha_i} < +\infty$. $i = 1, 2, \cdots, n$.and $\underline{\alpha} = diag(\underline{\alpha_1}, \underline{\alpha_2}, \cdots, \underline{\alpha_n})^{\mathrm{T}}$. $\overline{\alpha} = diag(\overline{\alpha_1}, \overline{\alpha_2}, \cdots, \overline{\alpha_n})^{\mathrm{T}}$.

Assumption (H2): Assume $h(0) = 0$ and there exist constant $0 < \gamma_i$ such that

$$\frac{h_i(x) - h_i(y)}{x - y} \geq \gamma_i,\ \forall x,\ y \in R,\ x \neq y,\ i = 1, \cdots, n.$$

Definition 1. SCGNNs (1) is said to be almost surely p-th moment ultimately bounded, if there exists a constant $K > 0$, for all initial value $\varphi(t)$ there exists a time T such that

$$\mathbb{E}|x(t, t_0, \varphi(t))|^p \leq K\ \ a.s.\ for\ \ t \geq t_0 + T.$$

Especially, when $p = 2$, it is said to be ultimately bounded in the mean square.

Definition 2. SCGNNs (1) is said to be global dissipativity in the mean square, if there exists a compact set $S \subset \mathbb{R}^n$, such that $\forall\varphi(t) \in L^2_{\mathscr{F}_0}([-\tau, 0]; \mathbb{R})$, $\exists T > 0$, when $t > t_0 + T$, $\mathbb{E}|x(t, t_0, \varphi(t))|^2 \subseteq S$, where $x(t, t_0, \varphi(t))$ denotes the trajectory of Eq.(1) from the initial state $\varphi(t)$. In this case, S is called a globally attractive set in the mean square. A set S is called a positive invariant in the mean square, if $\forall\varphi(t) \in L^2_{\mathscr{F}_0}([-\tau, 0]; \mathbb{R})$, $\mathbb{E}|\varphi(t)|^2 \subseteq S$ implies $\mathbb{E}|x(t, t_0, \varphi(t))|^2 \in S$ for $t \geq t_0$.

Lemma 1 [21]. For any $\varepsilon > 0$, $x \in R$, $y \in R$, the inequality $2xy \leq \varepsilon x^2 + \varepsilon^{-1}y^2$ holds.

Lemma 2 [5]. Let $a > 0$, $b > 0$, and $p > 1$, $q > 1$, and $\frac{1}{p} + \frac{1}{q} = 1$. Then we have the inequality $ab < \frac{1}{p}(a\varepsilon)^p + \frac{1}{q}(b\varepsilon^{-1})^q$, $\varepsilon > 0$ and the equality holds if and only if $(a\varepsilon)^p = (b\varepsilon^{-1})^q$.

Lemma 3. SCGNNs (1) is ultimately bounded in the mean square, if there are functions $V(t, x) \in C^{2,1}(\mathbb{R}_+\times\mathbb{R}^n; \mathbb{R}_+)$ and three positive constants c_1, k, λ satisfying

$$c_1|x(t)|^2 \leq V(t, x), \quad \mathscr{L}V(t, x) \leq k - \lambda V(t, x), \tag{3}$$

for all $(t, x) \in \mathbb{R}_+ \times \mathbb{R}^n$.

Proof. For any initial value x_0, let $x \triangleq x(t, t_0, \varphi(t))$, $x_0 \triangleq \varphi(t)$. By Itô's formula, we have

$e^{\lambda(t-t_0)}V(t, x) = V(t_0, x_0) + \int_{t_0}^t e^{\lambda(s-t_0)}[\lambda V(s, x) + \mathscr{L}V(s, x)]ds + \int_{t_0}^t e^{\lambda(t-t_0)}V_x(s, x)\sigma(t, x(t), x(t-\tau))d\omega(t)$.

From (3) and $\mathbb{E}\int_{t_0}^t e\lambda(t-t_0)V_x(s, x)\sigma(t, x(t), x(t-\tau))d\omega(t) = 0$, we can obtain

$$\begin{aligned} e^{\lambda(t-t_0)}\mathbb{E}V(t, x) &\leq V(t_0, x_0) + \mathbb{E}\int_{t_0}^t e^{\lambda(s-t_0)}kds \\ &\leq V(t_0, x_0) + \frac{k}{\lambda}\mathbb{E}e^{\lambda(t-t_0)}, \end{aligned}$$

then

$$\mathbb{E}|x|^2 \leq \frac{1}{c_1}\mathbb{E}V(t, x) \leq \frac{V(t_0, x_0)}{c_1}e^{-\lambda(t-t_0)} + \frac{k}{\lambda c_1}.$$

We can see that there exists $T > 0$ such that when $t \geq T + t_0$, $e^{-\lambda(t-t_0)}$ convergent to 0, we have

$$\mathbb{E}|x|^2 < \frac{k}{\lambda c_1}, \quad a.s.\ for\ \ t \geq t_0 + T.$$

So SCGNNs (1) is ultimately bounded in the mean square. □

3 Main Result

In this section, several sufficient conditions for the global dissipativity in the mean square of SCGNNs are established.