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Wind farm layout optimization under uncertainty

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A B S T R A C T

The optimal design of a wind farm with the aim of increasing the exploitation rate and power production is one of the challenging problems in the field of renewable energies. In this paper, the effect of using different hub height wind turbines in a farm on total power is studied. An exact mathematical formulation is presented in terms of the interaction matrix for multi-turbine wake effects considering different hub height wind turbines, and a new mixed-integer quadratic optimization model is developed. Then, the model is generalized to include the changes in wind characteristics and uncertain parameters. The model is solved using the linearization technique and an iterative method. The performance of the model and solution algorithm is evaluated by solving different problems borrowed from the literature. The computation results show the possibility of having a high-quality solution in a reasonable time in almost all cases.

1. Introduction

Renewable energy is generally defined as energy that comes from natural resources such as sunlight, wind, rain, and waves. This kind of energy is endorsed more than fossil fuel sources for its sustainability and compatibility with climate change and global warming. Wind energy which is used in power production is one of the oldest types of renewable energy. Electrical power is generated by the wind flow in an appropriately designed wind farm. Therefore, determining the optimal location of wind turbines has a significant impact on the performance of the wind farm and the total produced energy. The optimization of the layout of a wind farm is achieved by considering two main factors: the expected power output and the wake effect. The wake effect should be minimized in order to maximize the power output. It has two important features: (i) a decrease in the wind speed; (ii) an increase in the level of turbulence of wind flow. We consider this problem as the wind farm layout optimization (WFLO) problem.

2. Literature review

In the past decade, the wind turbine location problem was studied by many researchers. Mosetti et al. [1] solved the WFLO using a genetic algorithm to find the optimal number and location of wind turbines. They assumed a 10×10 grid of possible turbine locations used Jensen's wake decay model to analyze the wake effect under various wind speeds and directions. Improving the genetic algorithm and considering the Mosetti objective function, Grady et al. [10] achieved better results. Marmidis et al. [2] used Mossetti and Grady's objective function in a Monte Carlo simulation approach. Chowdhury et al. [3] determined the location of wind turbines considering the wind farm cost analysis based on the turbine rotor diameters and the number of turbines in the wind farm. They solved the wind farm model using the constrained Particle Swarm Optimization (PSO). Using a nested genetic algorithm, Chen et al. [4] optimized the layout of a 500m $\times$ 500m wind farm with different hub height wind turbines. They proved that the objective value has improved slightly more than the case with identical hub wind turbines. They also considered different cost models applied to WFLO with different hub height wind turbines, and demonstrated that the result can improve the cost per unit power of a wind farm in this case. A few mathematical programming approaches have been discussed in the literature. Donovan [5] proposed multiple mixed-integer linear models based on vertex packing problem between a couple of wind turbines. Truner et al. [6] developed a new mathematical framework to optimize the layout of a 10×10 grid of possible turbine location in a wind farm. They introduced interaction matrices which modelled the effect of wake interactions between the turbines. They used quadratic integer programming as well as mixed-integer linear programming

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approaches.

In this paper, we propose a mathematical programming framework which is a precise tool for modeling the WFLO problem in order to optimize the layout of a wind farm with different hub height wind turbines. In addition, we formulate an exact wake effect structure and compare the result with the layout in Refs. [6,4,1]. The main factors considered in this study are: (i) the number of wind turbines, (ii) wind directions and speeds, (iii) hub height of wind turbines, (iv) power output, and (v) the exact wake effect interaction. A quadratic integer programming and mixed-integer linear program are developed for different hub height wind turbines. Finally we present a heuristic algorithm with realistic results to solve the proposed models.

3. Wake model

When the wind hits a turbine, a cone of slower and more turbulent air is created behind it. This phenomenon, which is known as the wake effect in fluid-aerodynamics (see Fig. 1), has been investigated by researchers [7].

$$v = v_0\left[1 - \frac{2a}{\left(1 + \frac{\alpha D_{ij}^d}{r_1}\right)^2}\right] \tag{3.1}$$

$$r_1 = \alpha D_{ij}^d + r_r \tag{3.2}$$

$$\alpha = \frac{0.5}{\ln\left(\frac{z}{z_0}\right)} \tag{3.3}$$

$$a = 0.5\left(1 - \sqrt{1 - C_T}\right) \tag{3.4}$$

As depicted in Fig. 2, it is preferred to place the turbines in an area with two main characteristics: (i) the highest wind speeds, (ii) outside the wake of other turbines. With the assumption that the terrain is flat, and considering the information in Section 3.1, we use Jensen's wake decay model as follows:

In this study, we assume that the total kinetic energy deficit due to the multiple wake is equal to the sum of individual kinetic energy deficits generated by each wake [7,8]. For more details on the wake model, see Jensen [8]. With this assumption, we can reform Equation (3.1) to calculate the wind speed at position j after multiple sequences of turbines:

$$v_j = v_0\left[1 - \sqrt{\sum_{i \in J} d_{ij}^2}\right] \quad \text{Where} \quad d_{ij} = \frac{2a}{\left(1 + \frac{\alpha D_{ij}^d}{\alpha D_{ij}^d + r_r}\right)^2} \tag{3.5}$$

3.1. The nomenclature

All of the parameters involved in the following proposed models and wake model are defined in this section.

$\mathbb{J} = 1, \ldots, J$	Set of possible turbine locations $i \in \mathbb{J}, j \in \mathbb{J}$	N_T	Total number of wind turbines
$\mathbb{L} = 1, \ldots, L$	Set of wind directions $l \in \mathbb{L}$	z_k	Hub height of wind turbine of type k (m)
$\mathbb{N} = 1, \ldots, N$	Set of different velocities of free stream $n \in \mathbb{N}$	z_0	Surface roughness length (m)
$\mathbb{S} = 1, \ldots, S$	Set of scenarios, $s \in \mathbb{S}$	p^l	Probability of wind directions l
$\mathbb{K} = 1, \ldots, K$	Set of types of wind turbine, $k \in \mathbb{K}$	p^n	Probability of velocity of free stream n
v_0	Velocity of free stream (m/s)	p^s	Probability of scenario s
v_j	Wind velocity at position j (m/s)	m^l	Slope of line passing through the turbine for each direction
v	Downstream wind speed (m/s)	a	Axial induction factor
r_0	Turbine radius (m)	C_T	Trust coefficient
r_r	Downstream rotor radius (m)	α	Entrainment constant
r_1	Wake radius (m)	A	Swept area of wind blades (m^2)
ω_{ij}	Velocity loss in the wake at position j due to a turbine placed at position i	A_O	Overlapping area between wake and a turbine (m^2)
D_{ij}^d	Distance along the wind direction between turbines i and j (m)	P	Total power output (kW)
D_{ij}^v	Perpendicular distance along the wind direction between turbines i and j (m)	(X_i, Y_j)	Coordinates of each turbine (m)
D_{ij}^E	Euclidean distance between turbines i and j (m)	W_{ij}	Interaction matrix

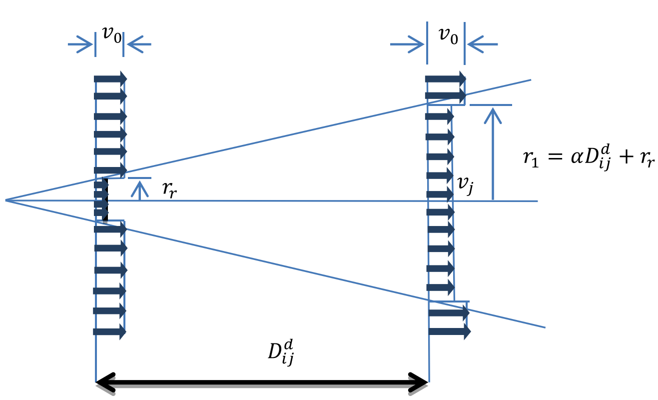

Fig. 1. Schematic representation of the wake effect.

4. Interaction matrix calculation

The exact structure of wake effect in a wind farm that determines which turbine is in the wake of the upstream turbine plays an important role in optimum wind turbine location. In addition, we need to take into account the wind direction in model formulation. In the present study, we propose a new method to define the exact mathematical structure of the wake effect in a wind farm by means of the interaction matrix concept. The dimensions of interaction matrices are related to the size of grid $(J \times J)$. Therefore, each element of the matrix shows the interaction between itself and $J-1$ other cells. We attribute a matrix to each direction of incoming wind using the following algorithm: