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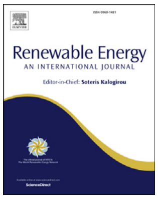

Prediction of multi-wake problems using an improved Jensen wake model

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ABSTRACT

The improved analytical wake model named as 2D_k Jensen model (which was proposed to overcome some shortcomes in the classical Jensen wake model) is applied and validated in this work for wind turbine multi-wake predictions. Different from the original Jensen model, this newly developed 2D_k Jensen model uses a cosine shape instead of the top-hat shape for the velocity deficit in the wake, and the wake decay rate as a variable that is related to the ambient turbulence as well as the rotor generated turbulence. Coupled with four different multi-wake combination models, the 2D_k Jensen model is assessed through (1) simulating two wakes interaction under full wake and partial wake conditions and (2) predicting the power production in the Horns Rev wind farm for different wake sectors around two different wind directions. Through comparisons with field measurements, results from Large Eddy Simulations (LES) as well as results from other commercial codes, it is found that the predictions obtained with the 2D_k Jensen model exhibit good to excellent agreements with experimental and LES data.

1. Introduction

Wind turbine wake effect, which is characterized by a significant deficit in wind speed and an increment of turbulence intensity, has been extensively studied in recent years. According to the work of [1] or [2], it was found that in large offshore wind farms, the average power loss due to wind turbine wakes is in the order of 10% (Middelgrunden wind farm) to 23% (Lillgrund wind farm) of the total energy production. For a wind energy project, accurate predictions of the wake flow behavior, such as the wind velocity distribution and the profile of turbulence intensity are quite important since the main work of the wind farm micro-sitting is to minimize the impact of upstream wakes on the power productivity of downstream turbines.

The wake models which are commonly used to predict turbine wake flows can be grouped into analytical and computational ones. At current stage, full LES with resolved boundary layer for a single wind turbine wake is still too expensive. An alternative approach is to use LES with Actuator Line (AL) method [3]. With the LES/AL method, it is possible to simulate multi-turbine wakes. However, these computational models are still too expensive as compared with the analytical models. So far they are not yet suitable for making wind farm parametric studies or layout optimizations. At present, simple analytical wake models are still the common tools to predict wind turbine wake effect. Especially in the case of wind farm layout optimization, the predictions of the wind turbines wake behavior need to be carried out for a large number of candidate farm layouts (necessary for optimization).

One of the most widely used and pioneering analytical wake models is the one proposed by Jensen [4] and later extended by Katic et al. [5]. The Jensen wake model (also named as Park model) has been extensively implemented in the literature [6,7] and the commercial software such as WAsP [8], GH WindFarmer [9] and WindPro [10]. The main features in this model are that the wake expands linearly with downstream distance, and the velocity deficit has a top-hat shape in the wake regions. Through a set of numerical experiments [1,11,12], the Jensen wake model has been proved to provide an acceptable prediction on the power losses within wind farm.

However, the Jensen wake model has two major limitations. The first one is that the assumption of a top-hat distribution of the wake

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deficit is far from reality and cannot describe the velocity field accurately. The Jensen wake model provides a simple velocity profile within the wake field, as it assumes that the wake velocity only changes in the streamwise direction and keeps constant in the cross-wind direction. From the space perspective, this simple model is known as 1D (one-dimensional) wake model. However, the self-similar Gaussian or trigonometric shape of the velocity deficit has been observed in wind tunnel measurements [13], data of operating wind farms [14] and numerical simulations [2]. According to these observations, it has been concluded that the wake velocity actually varies in two-dimensional space and Gaussian or trigonometric distribution is more appropriate to describe the velocity deficit in the far wake region.

The second limitation is the assumption of a constant wake decay parameter through the entire wake flow field. The wake decay coefficient k describes how fast or slow the wake expands by specifying the wake growth rate as it travels downstream. It is recommended in the WAsP Help Facility [15] to use $k = 0.075$ for offshore cases and 0.04 or 0.05 for onshore ones. Later in the literature [16], a semi-empirical expression $k = 0.5/\ln(h/z_0)$ was proposed for deriving the wake decay constant, where h is turbine's hub height and z_0 is the terrain surface roughness height. But in references [17–19], the authors pointed out that this semi-empirical formula is not adequate because it is only related to the ambient turbulence levels. In fact, the growth of the wake is governed by many more factors, in addition to the ambient turbulence, there are shear-generated turbulence and the turbulence created by the turbine. Therefore, the wake decay rate should not be a constant but a variable parameter that relating to the effective wake turbulence which is composed of ambient turbulence and turbine-added turbulence.

An improved wake model was proposed in Ref. [20] based on the classical Jensen (Park) wake model and two modifications: (1) a cosine shape distribution instead of the top-hat shape distribution for velocity deficit profiles in the wake; (2) the wake decay parameter k (constant for the Jensen wake model) depending on both the ambient turbulence and the wake-generated turbulence, as well as the downstream distance from the wind turbine. The performance of this new model (named as 2D_k Jensen model) was extensively investigated for the prediction of a single wake in our previous work [20]. The results showed that the 2D_k Jensen model can give a more physical and accurate description of the velocity deficit in the single wake. The main contribution in the present paper is to apply and validate the 2D_k Jensen model for the prediction of multi-wake effects downstream of a series of wind turbines.

The paper is organized as follows: in section 2, a brief introduction of the 2D_k Jensen model is presented. The applications of the 2D_k Jensen model to predict wake interactions between two wind turbines (with different downstream and crosswind displacements) operating at different inflow conditions are given in section 3. After that, the proposed model is adopted to investigate the effect of multiple wakes on power losses in a large offshore wind farm in section 4. Finally, the summary and conclusions are presented in section 5.

2. Improved Jensen wake model for multiple wakes prediction

2.1. 2D_k Jensen wake model

In order to give the reader a clear picture, a brief summary of the 2D_k Jensen model [20] is made first. The Jensen wake model is regarded as one of the most useful analytical wake models, which has been implemented in the widely-used commercial software. Based on the law of global momentum conservation, a mathematical expression is finally obtained to describe the wake velocity distribution downstream of a single wind turbine, which has the following form

$$u^* = u_0\left[1 - \frac{1-\sqrt{1-C_T}}{(1+kx/r_d)^2}\right] \tag{1}$$

where u^* is the wake velocity at a downwind position x, u_0 is the incoming wind speed, C_T is the wind turbine thrust coefficient, r_d is the rotor radius of the wind turbine, k is the semi-empirical wake decay constant which can be computed using

$$k = 0.5/ln(h/z_0) \tag{2}$$

where h is the hub height of the wind turbine, z_0 is the surface roughness height of the local terrain. The Jensen model describes the wake to be linearly expanding, and the spreading wake radius r_x at the downwind distance x is formulated as

$$r_x = r_d + kx \tag{3}$$

This original Jensen wake model is referred to as the one-dimensional Jensen model as it considers the wake speed changing only with downstream distance (as shown in Eq. (1)). This model is also known as the top-hat model since the shape of the predicted wake velocity along the cross-stream direction is like a top-hat. However, according to the classical theories and numerical computations of shear flows in the wakes of bluff bodies [21] as well as the wind tunnel investigations of the wake behind a single wind turbine [13], the wake velocity varies both in downwind and crosswind directions and it has an approximately Gaussian, cosine or polynomial shape in the crosswind direction after a certain downstream distance. Therefore, the Jensen wake model is too simple to describe the realistic distribution of wake deficit in the crosswind plane, a modification is highly needed to represent the physical wake velocity distribution.

In our previous work [20], the 2D_k Jensen model was proposed, in which the velocity profile in the cross section was represented by a cosine shape instead of a top-hat shape in the standard Jensen wake model. The expression of the developed 2D_k Jensen model is given as follows

$$u = (u_0 - u^*)\cos\left(\frac{\pi}{r_x}\cdot r + \pi\right) + u^* \tag{4}$$

where u^* is calculated using Eq. (1). It is worth noting that the proposed 2D_k Jensen model consists of two steps: the prediction of the original wake velocity at certain downwind position using the Jensen wake model; and then the redistribution of the calculated wake velocity along the crosswind direction using the cosine shape function.

For using the Jensen wake model, a crucial problem is to define the wake decay parameter k. The k value represents how the wake breaks down by specifying the growth of the wake width per unit length in the downwind direction. In the original Jensen model, the parameter k is supposed to be a constant with the value obtained from the semi-empirical function, Eq. (2) or some empirical instructions [8]. But researches in references [17–19] all showed that this approach is too realistic and not consistent with the ambient turbulence level, in which the roughness height is considered to be the only one influence factor on the wake decay parameter k. In fact, the wake growth is governed by the atmospheric turbulence as well as the turbulence generated by the rotor itself, this implies that instead of using a constant, the wake decay parameter k can be

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