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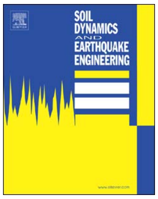

Random-effects regression model for shear wave velocity as a function of standard penetration test resistance, vertical effective stress, fines content, and plasticity index

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ABSTRACT

A random effect regression model is used to formulate a unified equation and estimate shear wave velocity (V_s). Standard penetration test (*SPT*) number, effective overburden pressure, plasticity index, and the fines content (F_c) are used as input parameters. First, a fixed model regression is used to obtain the regression parameters. *SPT* number and shear wave velocity are measured at 2 m intervals up to a depth of 10 m, for 71 boreholes, distributed evenly in Urmia city. Plasticity index and fines content are evaluated from laboratory tests that were performed on 355 samples obtained from the 71 boreholes (i.e., 5 samples from each borehole). Statistical analysis performed on the fixed effect model showed the need for examining the random effects arising from variable *SPT* test conditions in each borehole. A mixed effect regression model is employed to investigate such effects. The distribution of residuals is found to satisfy the normality criteria for the mixed effect model. A strong fit for the model is obtained, and through statistical evidence, it is implied that the proposed model is practical. The model's most prominent feature is the capability of unifying different soil types via the incorporation of plasticity index and fines content as inputs.

1. Introduction

Casualties and massive infrastructure damages indicate the urgent need for dynamic site characterization and robust models for seismic evaluation of a site's sustainability during natural hazards. Seismic characterization of a site is necessary to minimize damage caused by earthquakes. One of the oldest, yet efficient approaches of seismic characterization is performed by systematically obtaining statistical models that estimate the response of soil layers to earthquake excitations. The incorporation of a set of appropriate geotechnical properties plays a pivotal role in the efficiency of such models. In addition, these geotechnical variables should be correlated to a unique practical seismic parameter. Shear wave velocity is widely used for this purpose. The strong correlation of shear wave velocity with maximum shear modulus (G_{max}) of a soil is a great indicator of its importance in earthquake analysis. G_{max} can be correlated to the deformation potential of a given site during a seismic action.

Fig. 1 represents a typical modulus reduction curve that shows the rate of decrease in shear modulus with an increase in strain level. Several curves representing this relationship were created for different types of soils (e.g., [1–4]). For very small strain levels (i.e., approximately equal or less than 10^{-3}), the shear modulus of the soil is very close to the value of G_{max}. Therefore, employing an appropriate method to obtain the shear wave velocity of the soil for very small strains is necessary in seismic analysis. Once the shear wave velocity is obtained for very small strains, the small strain shear modulus can be computed as $G_{max} = \rho V_s^2$. In addition, V_s is directly used for ground motion prediction using next generation attenuation relations [5–9]. These relations employ V_{s30} as a required variable which is defined by Choi and Stewart [10] as the average V_s in the upper 30 m of the ground. Boore [11] proposed four methods to estimate V_{s30} for situations where data is not available for up to 30 m below the ground level. In general, when it comes to seismic analysis, V_s and G_{max} are the most important parameters employed in soil classification, liquefaction potential, and soil-structure interaction analysis [10].

There are three different approaches utilized for obtaining the shear wave velocity of soils. The first two approaches make use of laboratory and geophysical field measurements. The third approach, which is adopted in this article, aims at obtaining a robust correlation between shear wave velocity and simple geotechnical parameters (i.e., index properties) of a given soil.

Laboratory measurements of shear wave velocity require devices

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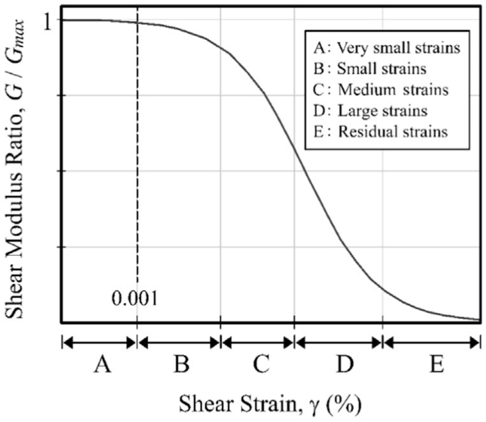

Fig. 1. Typical modulus reduction curve.

that are precise enough to measure the shear wave velocity at very small strain levels. For example, resonant column test is used to obtain shear wave velocity from the resonant frequency and the weight and dimensions of specimens [12–15]. Bender elements and shear plates are two types of piezoelectric transducers that are used to obtain the shear wave velocity from the distance, between the two transducers located at two ends of a specimen, and the wave travelling time [16]. Piezoelectric transducers are accommodated in a cyclic triaxial apparatus combined with precise axial strain measurement devices to obtain the shear wave velocity. The accuracy of the results for laboratory measurements is highly sensitive to sample disturbance. During sampling, the weak boundaries between soil particles are broken and some level of disturbance occurs. Since the effect of sampling disturbance on the stiffness of the soil is remarkable for low strain laboratory tests, accurate results for shear wave velocity measurements are not possible unless expensive freezing techniques are used [17].

Seismic geophysical field tests are the most reliable methods to obtain the shear wave velocity for a soil at various depths. Crosshole test (CHT) [18], downhole test (DHT) [19], seismic cone penetration tests (SCPTs) [20,21], multichannel analysis of surface waves (MASW) [22], and spectral analysis of surface waves (SASW) [23] are well established methods for V_s measurement at very small strain levels. Although these tests are performed in low disturbance conditions, various restrictions such as space, cost, and noise limit their universal utilization. In addition, field tests have proven to be rather expensive and time consuming. The aforementioned drawbacks of the laboratory and field tests led to the development of a new approach in which statistical methods are used to correlate shear wave velocity and simple geotechnical parameters, such as the index properties of a soil.

Statistical approaches are powerful tools often used to find correlations between shear wave velocity and geotechnical parameters of soil. Most studies in this area attempted to correlate V_s with *SPT* blow counts (*N*), directly [24–33]. Table 1 summarizes some of the empirical relationships suggested to estimate V_s from SPT number (*N*) and depth (*D*) of the soil.

Hara [34] found that dynamic Poisson's ratios were insignificantly influenced by the change in Young's moduli when axial strains were in the order of 10^{-3}. The very first attempt to present a relationship between shear moduli and *SPT* N-value of the soil was made by Kanai et al. [35], where they introduced two linear boundaries for the relationship between shear modulus and *SPT* values for clay and sand. Since their pioneering work, other researchers have attempted to obtain similar correlations for different types of soils. Imai and Yoshimura [26,27] correlated the mechanical properties of soils to the primary and secondary body wave velocities. They directly measured S- and P-wave velocities using a PS logging system. They acquired the standard penetration test resistance, shear wave velocity, and unconfined compressive strength of soil samples from 242 boreholes that were distributed all over Japan. They proposed three separate equations for three types of soils and correlated the *SPT* blow count, *N*, to the shear wave velocity of clay, sand, and silt. It was the first time that the shear wave velocity of soil was correlated to *N* using an exponential form (i.e., $V_s = AN^B$).

Ohba and Toriumi [31] and Ohta et al. [39] obtained same-type equations for alluvial soil deposits including sandy, clayey, and their alternate layers. Since then, the work of these researchers have been followed by others in an attempt to obtain similar correlations for different types of soils. Seed and Idriss [1,32] proposed a simple equation that correlates shear wave velocity of a soil to *SPT* values. Using statistical analysis, Lee [36] presented numerous regression models that estimate shear wave velocity from *SPT* resistance, depth, effective overburden pressure, and soil type. The effect of *SPT* number (*N*) and soil type on shear wave velocity of the soil has been studied by Iyisan [28]. The study showed that the same *N* values result in the same V_s for different type of soils, with an exception of gravel. Hasançebi and Ulusay [25] presented several equations for V_s versus *SPT N* number by using 97 sets of data gathered from the Northwest area of Turkey.

Different equations were obtained for clayey and sandy soils. Using datasets gathered from seismic micro zonation studies in India, Anbazhagan and Sitharam [37] presented an equation to determine the shear wave velocity of the soil based on the modified standard penetration test (N_{1-60}). Brandenburg et al. [38] used statistical regression analysis and presented an equation to estimate V_s for soils under Caltrans bridges. Gathering datasets from 79 logs in 21 bridges, they correlated the natural logarithm of V_s (i.e., $Ln(V_s)$) with *SPT N* number and effective overburden pressure for sandy, silty, and clayey soils. Using datasets gathered from Taiwan, Kuo et al. [30] presented an equation to determine V_s based on SPT *N* number and depth. Ghorbani et al. [17] presented an equation to estimate the shear wave velocity from the modified *SPT N* number, N_{1-60}, and effective overburden pressure. They employed polynomial neural networks for their model and used datasets from different zones of the world.

The effect of parameters like *SPT*, effective overburden pressure, the percentage of fine grains, depth, and tip resistance in the cone penetration test for the shear wave velocity of soils have been studied by multiple researchers [e.g., 24, 25, 28, 38]. The outcome from different studies performed in the past has been diverse. Several studies have revealed that effective overburden pressure, porosity, and geological age influence the value of G_{max} for different soils. Others have reported that pre-consolidation stress has a negligible effect on G_{max} [4,40–43]. The effect of plasticity index (*PI*), on shear wave velocity of soils, remained controversial. Some studies showed a direct relationship between G_{max} and *PI* (e.g., [4,41,42]) while reverse relationships were reported by others (e.g., [4,41,42,44]). Hardin and Drnevich [41] showed that the most influential parameters in the evaluation of G_{max} and V_s of soils are unit weight, porosity, and effective pressure. Age and cementation have been found to have little or no effect on G_{max} and V_s, depending on the type of soil. Direct relationships between V_s and vertical effective pressure, age, cementation, and the pre-consolidation stress have been obtained by Dobry and Vucetic [45]. They also showed that a reverse relationship governs the correlation between V_s and porosity.

By reviewing the existing literature, it can be said that a one-parameter linear equation is not capable of correlating shear wave velocity and the index properties of a soil. In addition, the scattered datasets and the associated very-weak trend lines suggest the importance of including the effect of parameters other than the *SPT N* number. On the other hand, the wide variety of soil types makes it difficult to define variables that have strong correlation with shear wave velocity. Nonetheless, efforts should continue to be made to obtain correlations that adequately estimate the seismic behavior of soils, while limiting the utilization of existent correlations to the estimation of the need for seismic consideration.

Some researchers have considered depth, as an input parameter for