Soil Dynamics and Earthquake Engineering 100 (2017) 296–300

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

Three-dimensional shakedown analysis of ballasted railway structures under moving surface loads with different load distributions

Yan Zhuang[a,*], Kangyu Wang[b]

[a] Key Laboratory of Ministry of Education for Geomechanics and Embankment Engineering, Geotechnical Research Institute, Hohai University, 1 Xi Kang Road, Nanjing 210098, China

[b] College of Civil and Transportation Engineering, Hohai University, 1 Xi Kang Road, Nanjing 210098, China

ARTICLE INFO

Keywords:
Melan's shakedown theorem
Shakedown analysis
Railway structures
Moving surface loads
Numerical investigation

ABSTRACT

Shakedown of ballasted railway structures was analyzed based on Melan's shakedown theorem, in which the wheel/rail contact was approximated by Hertz (circular and elliptical contact areas), uniform and trapezoidal load distributions, separately. The shakedown solutions incorporating to the three-dimensional finite element model calculated shakedown multiplier by means of a self-equilibrated critical residual stress field. The shakedown multiplier for multi-layered ballasted railway structure was determined as the minimum one among all layers. The results showed that elliptical Hertz and uniform load yielded the largest and smallest shakedown limits, respectively, with the maximum difference of approximately 64%. The shakedown limits always occurred at ballast layer for relatively small frictional coefficient, whilst occurred at rail for low rail's yield stress with large frictional coefficient. As expected, the shakedown limits decreased as the ballast stiffness and thickness increased, especially for relatively small frictional coefficient; while increased with raising rail's yield stress. The material properties and thickness should therefore be optimally designed so as to provide a maximum resistance to the structure failure and reduce the material costs.

1. Introduction

The evaluation of railway performance requires proper assessment of the permanent deformation and fatigue under moving traffic loads. A particularly effective way of gaining insight into the combination of loads at which the structure shakedown is to evaluate the shakedown limit loads using either static [1] or kinematic shakedown theorem [2]. Shakedown limits have been recognized as the rational design criterion since the 1960's for metallic contacts such as rails, roller bearings, and traction drives [3]. Sharp and Booker [4] were among the first to apply shakedown theory to the stability analysis of soil structures, in which a semi-analytical approach for determining the shakedown loads was developed. The shakedown problem has been treated numerically using a combination of finite elements (FE) and linear or nonlinear programming techniques [5–10], however, this may lead to nonlinear convex optimization problems, which are typically characterized by large numbers of unknowns and constraints when problems of practical relevance are considered.

Shakedown theory has also received much attention from researchers in the field of railway engineering, however, the research are mainly focused on the rolling contact fatigue between wheels and rails [11–14]. As a result, the shakedown analysis for the substructures of the railway is very limited in the literature, especially taking into account the layered behavior of the railway structures, and therefore deserves more research attention.

This paper investigates shakedown of the ballasted railway structures under moving surface loads based on Melan's shakedown theorem. The wheel/rail contact is assumed to be a three-dimensional (3D) situation, and is approximated by Hertz (circular and elliptical contact areas), uniform and trapezoidal load distributions, separately. The analytical shakedown solutions are developed incorporating to the 3D FE model, based on which the influence of frictional coefficient, rail's yield stress, elastic modulus of ballast, and the thickness ratio of ballast to subballast on the shakedown limits are presented and discussed.

2. Formulation of the problem

2.1. Description of the wheel/rail contact

The rolling and sliding contact between wheel and rail was usually approximated by Hertz, uniform and trapezoidal load distributions, as presented in Fig. 1, in which P is the total normal load applied in the

* Corresponding author.
E-mail addresses: zhuangyan4444@hotmail.com (Y. Zhuang), kangyu02@163.com (K. Wang).

http://dx.doi.org/10.1016/j.soildyn.2017.06.012
Received 29 May 2017; Received in revised form 12 June 2017; Accepted 13 June 2017

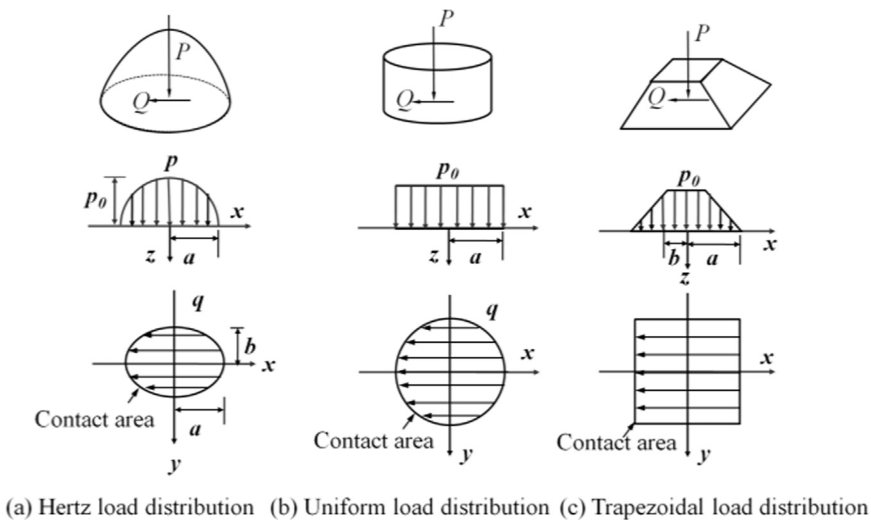

Fig. 1. Different load distributions for the wheel/rail contact.

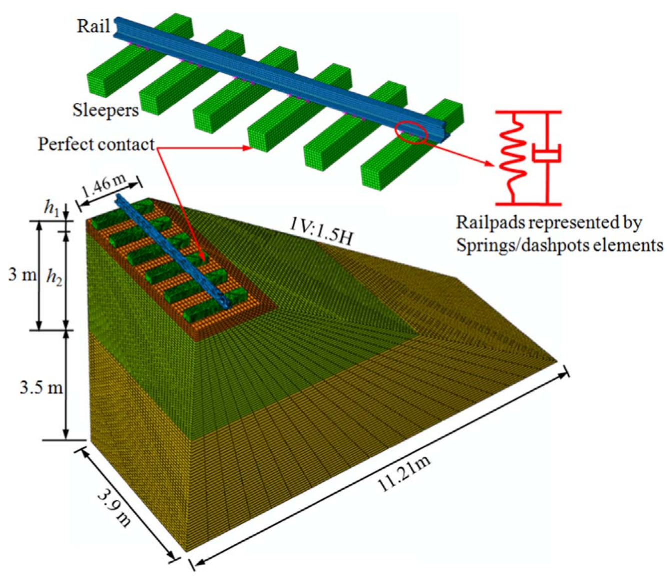

Fig. 2. FE mesh of the fully 3D ballasted railway structures.

vertical direction, and Q is the total shear load applied in the travel direction. The normal and shear loads are correlated by the frictional coefficient μ as:

$$Q = \mu P \tag{1}$$

As for Hertz load distribution, the contact area between wheel and rail is either circular or elliptical, which depends heavily on the magnitude of the load. The 3D elliptical Hertz load has a maximum compressive pressure $p_0 = 3P/(2\pi ab)$ at the load center ($x = y = z = 0$), in which a and b are the major and minor axis of the contact area. In this research, the aspect ratio of the elliptical contact area b/a was fixed as 0.5, and $a = 10$ mm [11,15]. It should be noted that the circular contact area is a special case of the elliptical contact area, i.e., $a = b$. For the trapezoidal load distribution, the value of b/a was fixed as 0.5, which was the same as that in Zhao et al. [16].

2.2. Analytical shakedown solutions

For the 3D problem considered herein, Yu [17] assumed that under a moving surface load, the most critical plane was one of the xz plane defined by y = constant. On these planes, the only critical non-zero residual stress that may increase the shakedown limits would be σ_{xx}^r, as a function of y and z. In the y-direction, the residual stress σ_{yy}^r may well exist, as a function of z.

For Mohr-Coulomb material, the shakedown multiplier requires that [17]:

$$\lambda \le \frac{c}{|\sigma_{xz}^e| + \sigma_{zz}^e tan\phi} \tag{2}$$

For material obeying Von-Mises criterion, the shakedown multiplier can be given by [18]:

$$\lambda \le \frac{\sigma_{0.2}}{\sqrt{3}\,|\sigma_{xz}^e|} \tag{3}$$

The shakedown solutions can be applied to layered materials to obtain the maximum shakedown limit parameter λ_{sd}^1, λ_{sd}^2 … λ_{sd}^n respectively. To avoid plastic flow with each passage of the load in any element of substrate, layer or interface, the maximum shakedown limit parameter λ sd must be below the minimum value of any of these shakedown limit parameters, i.e.,

$$\lambda_{sd} = min(\lambda_{sd}^1, \lambda_{sd}^2, \lambda_{sd}^3, ..., \lambda_{sd}^m) \tag{4}$$

The elastic stress fields in the layered system are much more complicated than those in a homogeneous half-space, and have not yet been given by any closed form expression. Therefore, FE analyses for elastic stress fields are carried out by means of the FE software ABAQUS.

3. Shakedown analysis of ballasted railway structures

3.1. Description of the FE models

A 3D FE model was developed, which included the rail, railpad, sleeper and the subsystem, as shown in Fig. 2. The rail (CHN 60 kg/m) was 176 mm high with the cross section of 7.745 × 105 mm^2, and was discretely supported by the sleepers with spacing of 0.65 m. The 10 mm thick railpad was represented by springs/dashpots element with the vertical stiffness of 150 MN/m and viscous damping of 13.5 kNs/m [19] as a connection between rail and sleeper. Each sleeper was formed from prestressed concrete with dimensions 0.2 m × 0.2 m × 2.6 m. The 6.5 m subsystem consisted of ballast, suballast and subgrade layers Due to symmetry, only half of the railway structures were modeled. The normal and shear pressures for different load distributions were applied on the top surface of the rail. The elastic-plastic materials were assumed, with the properties and thickness shown in Tables 1–3 [20]. The FE model was discredited by a total of 537 thousands eight-noded, reduced-integrated, brick elements (C3D8R). In order to present accuracy FE results, the full 3D model was also developed as a validation, whose maximum difference with the symmetrical model was within 4%.