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A novel method for identifying surface waves in periodic structures

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ABSTRACT

In this study, the propagation of surface waves in both one- and two-dimensional periodic structures is investigated. By combining finite element method, an energy distribution parameter is defined and a new method for identifying surface wave modes is suggested. The effectiveness of this new method is validated by comparing with some related studies. Furthermore, this method is used to study a two-dimensional periodic pile-soil system based on a three-dimensional numerical model and the dispersion curves of surface waves are easily obtained. To show the efficiency of attenuation zone, the responses of a finite periodic pile-soil system to a surface wave input are simulated. Results demonstrate that the region of excitation frequency in which vibration reduction occurs is fully consistent with the theoretical attenuation zone for surface waves. The advantage of this method is that it makes the study of surface waves more convenient and accurate.

1. Introduction

In the past two decades, the propagation of elastic waves in periodic structures has attracted a lot of attention due to its unique properties such as attenuation zones (AZs) and negative refraction [1,2]. These characteristics lead to some potential applications such as noise control and seismic isolation [3–5]. However, compared with bulk waves, less attention has been paid to the propagation of surface waves (SWs). SWs can be described as non-dispersive waves which propagate near the free surface of a homogenous solid medium. The propagation of SWs in non-homogenous systems such as periodic structures shows different characteristics. For example, the phase velocity and group velocity of SWs vary with frequency, i.e., the SWs in periodic structures are dispersive waves. Furthermore, some theoretical and experimental studies reported that, similar with bulk waves, AZs also exist when SWs propagate in periodic systems [6–8].

There are some studies concerning the dispersion relation of SWs propagating in periodic structures. Djafari-Rouhani et al. [9] presented a detailed procedure of plane wave expansion (PWE) method and investigated the dispersion curves and AZs of Rayleigh waves in periodic alternating layers of two elastic and isotropic materials. Using PWE method as well, Tanaka et al. [10] reported SWs as well as pseudo SWs in two-dimensional periodic elastic structures consisted of AlAs cylinder inclusions embedded in GaAs background. Sun and Wu [11] studied the SWs propagating in two-dimensional steel/epoxy periodic structures by finite difference time domain (FDTD) method. Yan and Wang [12] developed a wavelet-based method to calculate the dispersion curves of SWs in two-dimensional periodic structures including both mixed fluid/solid systems and solid/solid systems. By finite element method combined with sound cone limitation, i.e., surface wave modes only appear below the sound line, Assouar et al. [13] studied SWs propagating in two-dimensional periodic structures with three different types: fluid/solid, solid/solid, and air connected stubbed substrate, respectively. Based on finite element method as well, Graczykowski et al. [14] proposed the center approach of elastic energy to discuss true and pseudo SWs in one-dimensional periodic structures. The transmission, reflection, and surface-to-bulk losses in a finite periodic system were also calculated.

Though some important achievements have been obtained on SWs propagating in periodic structures, there are some weaknesses in the previous methods. For example, the cone of sound criterion is able to sort most of surface modes, however, it fails in the case that some surface modes appear in the radiative region [14]. The aim of this study is to propose an efficient and accurate method for identifying surface wave modes from bulk wave modes. Investigation shows that the dispersion curves of SWs propagating in periodic structures can be easily obtained by this method. Moreover, the present method is still reliable when dealing with the periodic systems with different dimensions and with diverse geometries. In fact, pile barriers often used in ambient engineering can be considered as a periodic structure. The method proposed in this study is able to analyze vibration screening by periodic pile barriers according to the AZ of periodic structures. Therefore, this study provides a new perspective for SWs isolation by periodic structures in practical engineering.

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2. Basic theory

Assuming continuous, perfectly elastic, small deformation and without consideration of damping, the governing equation of waves propagating in periodic systems can be given as

$$\rho\frac{\partial^2\mathbf{u}}{\partial t^2} - \nabla\cdot c\nabla\mathbf{u} = 0 \tag{1}$$

where ρ is the mass density, $\mathbf{u}$ is the displacement vector, t is time parameter, ∇ is differential operator, and c is the elastic constant.

According to the theory of periodic structures, the displacement field can be expressed as

$$\mathbf{u}(\mathbf{r}, t) = e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\mathbf{u}_{\mathbf{k}}(\mathbf{r}) \tag{2}$$

where $\mathbf{r}$ denotes the coordinate vector; $\mathbf{k}$ is the reduced wave vector; ω is angular frequency; and $\mathbf{u}_{\mathbf{k}}(\mathbf{r})$ is a periodic function about the periodic constant $\mathbf{R}$. Thus, $\mathbf{u}_{\mathbf{k}}(\mathbf{r})$ can be written as

$$\mathbf{u}_{\mathbf{k}}(\mathbf{r}+\mathbf{R}) = \mathbf{u}_{\mathbf{k}}(\mathbf{r}) \tag{3}$$

Substituting Eq. (3) into Eq. (2), the periodic boundary condition is obtained,

$$\mathbf{u}(\mathbf{r}+\mathbf{R}, t) = e^{i\mathbf{k}\cdot\mathbf{R}}\mathbf{u}(\mathbf{r}, t) \tag{4}$$

In fact, a periodic structure can be replaced by a unit cell in terms of the periodic boundary condition of Eq. (4). Basically, to deal with the problem of SWs, a semi-infinite system with free boundary condition should be considered. Because the displacements of SWs decay rapidly along the depth, to simplify the analysis, a unit cell with large parameter h about depth and a fixed boundary on the bottom is usually used to replace the unit cell with infinite thickness [13].

Then, the eigenvalue equation for the unit cell can be written as

$$(\mathbf{\Omega}(\mathbf{k}) - \omega^2\mathbf{M})\cdot\mathbf{u} = \mathbf{0} \tag{5}$$

where the stiffness matrix $\mathbf{\Omega}$ is a function of the wave vector, $\mathbf{M}$ is the mass matrix. Taking the periodic boundary condition Eq. (4) and fixed boundary condition of the unit cell (i.e. $\mathbf{u}=\mathbf{0}$) into account, the eigenvalues of Eq. (5) can be obtained by a commercial software (COMSOL Multiphysics 5.1) for every given reduced wave vector $\mathbf{k}$.

However, surface modes as well as bulk modes mix in the dispersion curves of the foregoing numerical model. To distinguish the surface modes from the bulk modes, an energy distribution parameter ξ is proposed in this study, which reflects the energy distribution of a considered wave mode in this system:

$$\xi = \frac{\int_{S(2\lambda)} W_e ds}{\int_{S(h)} W_e ds} \tag{6}$$

in which the integrals are calculated in the area or volume of the unit cell with height of 2λ and h, respectively; The symbol λ is the wavelength of a considered wave and W_e is the elastic strain energy density (ESED), respectively. Eq. (6) demonstrates that the parameter ξ varies from 0 to 1. Obviously, the waves are surface modes when ξ approaches to 1. In the present study, it is suggested that the criterion for identifying surface modes is $\xi > 0.9$. In fact, the energy distribution parameter defined in Eq. (6) has a clear physical meaning. Its magnitude represents how much energy is located within twice the wavelength of a considered wave mode. When the amplitude of displacement decays exponentially along depth away from free surface, the wave is considered as a surface wave. In homogenous media, analytical solutions for surface waves can be found. However, there is no analytical solution to surface waves for periodic structures up to now. By defining an energy distribution parameter, a new criterion is proposed and extended to the periodic system to identify surface waves. The effectiveness of the present approach is validated by comparing with some related results. Furthermore, the dynamic responses of a finite periodic pile-soil system are studied to show the efficiency of the present method in engineering vibration reduction.

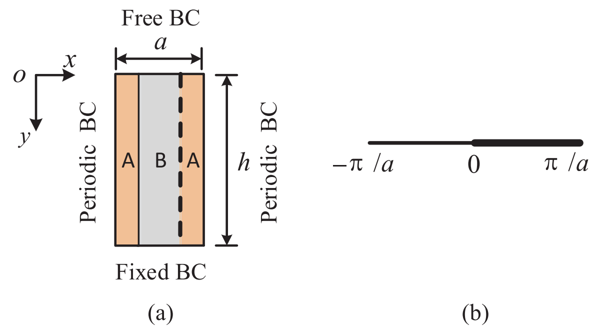

Fig. 1. Numerical model for the one-dimensional periodic structure: (a) unit cell, (b) the reduced wave vector.

3. Results and discussion

The problem of SWs propagating in a one-dimensional periodic structure reported by Hu et al. [15] is reconsidered by using the present method. Layered elastic material copper and aluminum are placed alternatively in the x direction to form a periodic half-space. The mass density ρ, Poisson ratio ν and Young modulus E of copper are taken as 8920 kg/m^3, 0.355 and 115 GPa, respectively; and those of aluminum are 2700 kg/m^3, 0.334 and 69 GPa, respectively. Due to periodicity of the structure, only a unit cell is considered as shown in Fig. 1(a). The y axis is chosen to be perpendicular to the free surface; a denotes the periodic constant of the unit cell. To check the existence of SWs, the depth should be large enough, i.e., larger than several periodic constants. In most of the previous literatures [16,17], it is taken as $h\geq 10a$. In the present study, the unit cell with a depth $h=20a$ is considered.

For the unit cell mentioned in Fig. 1, Fig. 2(a) gives the dispersion curves of SWs obtained by the present method. The results obtained by Hu et al. are also plotted in Fig. 2(a) for comparison. The lower and upper branches of the dispersion curves are opened at the reduced wave number $k = \pi/a$, where the group velocity of SWs is equal to zero. The AZ for SWs is the region between the two branches. Good agreement is found for this case. Furthermore, choosing the two materials as a same material such as copper, the copper-aluminum system becomes a half-space of homogenous medium which has analytical solutions for SWs [18]. In this case, the SWs are non-dispersive and there are no AZs existing in the isotropic homogenous system. The results are shown in Fig. 2(b) and good agreement is also found. It is obvious that the present method is correct.

In order to depict the decaying characteristics of SWs vividly, the normalized ESED and normalized displacement amplitude of surface modes changing with depth along the interface denoted by dash line in Fig. 1(a) are plotted in Fig. 3(a) and (b), respectively. Points s_1 and s_2 in Fig. 2(a) correspond to lower and upper branches of the dispersion curves of SWs, respectively. The symbols u_x and u_y in Fig. 3(b) denote the component of displacement in the x and y directions, respectively. It is obvious that both elastic strain energy and displacements of the considered waves are concentrated in a very thin layer near the free surface, and decrease rapidly from a distance far away the free surface, which indicates that the considered waves are indeed SWs.

As another example, the dispersion relation of SWs in a pile-soil periodic system is investigated by using three-dimensional (3D) numerical model combining with the identifying criterion proposed in the present study. The system is formed by foam piles embedded in soil substrate. The two materials are assumed to be elastic and isotropic. The mass density ρ, Poisson ratio ν and Young modulus E of foam are taken as 60 kg/m^3, 0.32 and 37 MPa, respectively; and those of soil as 1800 kg/m^3, 0.35 and 20 MPa, respectively. The unit cell is shown in Fig. 4(a). The periodic constant of the unit, the radius of pile and the