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Dynamic variability response functions for stochastic wave propagation in soils

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ABSTRACT

In this paper, shear wave propagation in soils is examined in a stochastic context considering spatial variability of the shear modulus soil parameter. To this purpose, the recently established concept of dynamic mean and variability response functions (DMRF, DVRF) is reformulated in the framework of stochastic finite element analyses of shear wave propagation problems in order to efficiently calculate the response time history statistics of the soil surface. Similarly to the approximation formulas of classical VRFs, a fast Monte Carlo simulation procedure is implemented to numerically evaluate the above functions in the time domain. The main advantage of the proposed methodology lies on the independence of the DMRF and DVRF on the marginal probability density function and correlation structure of the stochastic system parameter, which in our case is assumed to be the inverse of the soil shear modulus $1/G$. By integrating the product of the spectral density of $1/G$ with the DMRF and DVRF, the mean and variance of the ground response are obtained at each time step of the dynamic analysis. The method also allows for the estimation of time dependent but spectral and probability distribution free upper bounds of the response mean and variance. To illustrate the efficiency and applicability of the proposed approach, stochastic finite element analyses of wave propagation of a Ricker synthetic wavelet as well as a recorded earthquake motion in 1D and 2D soil domains are performed and a sensitivity analysis is carried out with respect to various correlation structures of the underlying random fields representing $1/G$. The accuracy of the proposed methodology is validated with comparison to direct Monte Carlo simulation. Useful conclusions regarding the sensitivity of the system response to the spectral characteristics of the underlying random fields representing $1/G$ are drawn.

1. Introduction

In recent years, numerical methods that incorporate stochastic material properties of soils in various geotechnical engineering applications have gained considerable attention. Characteristic applications include footing bearing capacity on soils with spatially variable cohesion and friction angle [1,2], footing settlement analyses for soil layers with stochastic elasticity modulus [3,4]. In addition, consolidation of soil layers with uncertain properties [5], as well as seepage analyses [6] were investigated, while slope stability analyses have been carried out [7]. Furthermore, dynamic problems with stochastic soil parameters are investigated in a number of research articles [8–11]. A wide range of geotechnical engineering applications where the soil uncertain material properties are taken into account can be found in [12]. All the above test cases have been analyzed in the context of the finite element method incorporating in various ways the stochastic property of the soil. Most of the studies regarding uncertain soil properties are based on some version of the globally applicable Monte Carlo Simulation (MCS) method. The advantage of the MCS is its applicability to any probabilistic finite element model regardless of its complexity. Besides the well-known limitations of MCS due to its large computational cost, the main disadvantage of this approach is that the correlation structure of the underlying stochastic property of the soil materials has to be known, which is rarely the case. Thus, the study of sensitivity of the required response with respect to different correlation characteristics makes the MCS almost prohibitive for the treatment of realistic examples.

In order to tackle the aforementioned limitations, the concept of variability response functions (VRF) was introduced in a number of articles [13–15]. The VRF is a Green's function which relates the variance of a response quantity of a system to the spectral density function of its underlying uncertain parameters [16]. The VRF depends on deterministic system properties related to geometry, boundary and loading conditions, mean material properties, as well as the standard

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deviation σ_{ff} of the considered stochastic parameter. The VRF was initially expressed in closed form for statically determinate and indeterminate beam and truss structures under deterministic loading conditions. Later the VRF concept was extended to stochastic plate bending problems [17]. As stated in [18], in most problems, a closed form expression of the VRF is extremely difficult if not impossible to extract. The VRF can alternatively be estimated numerically using a so-called fast Monte Carlo finite element based procedure explained in [18,19]. Other applications of the variability response function include the study of apparent material properties for heterogeneous random materials [16] as well as robust design optimization taking into account the stochastic system parameters [20]. The standard VRF was formulated for static stochastic problems. An extension of the VRF for dynamic problems leading to the dynamic variability response function (DVRF) was introduced in [21]. The DVRF and the closely related Dynamic Mean Response Function (DMRF) provide the same spectral-free advantages of the VRF for dynamic loadings and give insight on the sensitivity of the response of dynamical systems with respect to the stochastic properties [22].

In this paper, the concept of DVRF and DMRF is used to simulate the shear wave propagation in soils with spatially varying shear modulus G. The independence assumption of DVRF, DMRF functions to the spectral density function of the underlying material property makes the methodology ideal for problems involving soil materials where lack of sufficient data is the common case. It is shown that through the DMRF, DVRF functions, the time history of mean and variance of the response quantity of interest can be accurately and efficiently calculated for stochastic shear wave propagation problems in 1D and 2D soil domains. A fast Monte Carlo Simulation (FMCS) method is used in order to numerically evaluate the DMRF and DVRF functions for displacement, velocity and acceleration of the soil layer surface. Application of finite element analyses of propagation of synthetic Ricker wavelets, as well as a real recorded earthquake motion are used as test-cases to demonstrate the power of the method. Furthermore, upper bounds of the mean and variance of the response quantities of interest are established through the use of the calculated DMRF and DVRF functions. The accuracy of the proposed approach is proven by direct comparison of the results obtained via the MCS method. Useful conclusions regarding the sensitivity of the statistical characteristics of the soil response on the underlying nature of the material correlation properties are drawn.

2. Simulation of stochastic shear wave propagation in soils

In this work, the stochastic soil parameter considered is the inverse of the soil shear modulus $1/G$ which varies randomly along the vertical axis y in 1D models and in the horizontal and vertical axes x, y for plane strain models. In the general 2D, case the following relation holds:

$$\frac{1}{G(x, y)} = F_0 \cdot (1 + f(x, y)) \tag{1}$$

where $G(x, y)$ denotes the soil shear modulus at the spatial point with coordinates (x,y), $f(x, y)$ is a zero-mean homogeneous stochastic field which models the spatial variation of $1/G$ around its mean value $F_0 = 1/G_0$.

In wave propagation analyses in the time domain, the general dynamic equilibrium system of equations has to be solved:

$$\boldsymbol{M}\ddot{\boldsymbol{u}}(t) + \boldsymbol{C}\dot{\boldsymbol{u}}(t) + \boldsymbol{K}\boldsymbol{u}(t) = \boldsymbol{P}(t) \tag{2}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the mass, damping and stiffness matrices, $\ddot{\boldsymbol{u}}(t)$, $\dot{\boldsymbol{u}}(t)$ and $\boldsymbol{u}(t)$ are the acceleration, velocity and displacement vectors and $\boldsymbol{P}(t)$ is the external force vector.

In order to integrate the equation of motion (2), the implicit unconditionally stable α-method (Hilber-Hughes-Taylor) [23] is used. The basic parameters of the method are the timestep Δt and the parameter α which lies in the interval $[-1/3, 0]$ and controls the numerical damping. The two other parameters β and γ are calculated as a function of α by the following relations:

$$\beta = \frac{1}{4}(1 - \alpha)^2 \gamma = \frac{1}{2}(1 - 2\alpha) \tag{3}$$

According to this time integration scheme, the following relations are used:

$$\begin{aligned} \dot{\boldsymbol{u}}_{n+1} &= \dot{\boldsymbol{u}}_n + [(1-\gamma)\ddot{\boldsymbol{u}}_n + \gamma\ddot{\boldsymbol{u}}_{n+1}]\Delta t \\ &= \dot{\boldsymbol{u}}_n + \ddot{\boldsymbol{u}}_n\Delta t + (\ddot{\boldsymbol{u}}_{n+1} - \ddot{\boldsymbol{u}}_n)\gamma\Delta t \end{aligned} \tag{4}$$

$$\begin{aligned} \boldsymbol{u}_{n+1} &= \boldsymbol{u}_n + \dot{\boldsymbol{u}}_n\Delta t + [(1-2\beta)\ddot{\boldsymbol{u}}_n + 2\beta\ddot{\boldsymbol{u}}_{n+1}]\tfrac{1}{2}\Delta t^2 \\ &= \boldsymbol{u}_n + \dot{\boldsymbol{u}}_n\Delta t + \ddot{\boldsymbol{u}}_n\tfrac{1}{2}\Delta t^2 + (\ddot{\boldsymbol{u}}_{n+1} - \ddot{\boldsymbol{u}}_n)\beta\Delta t^2 \end{aligned} \tag{5}$$

In Eqs. (4) and (5) the quantities with subscript n refer to time t while $n + 1$ refers to time $t + \Delta t$ for a chosen timestep Δt. Combining Eqs. (4) and (5) so that the basic unknown is $\boldsymbol{u}_{n+1}$, the following relations hold:

$$\ddot{\boldsymbol{u}}_{n+1} = \left(1 - \frac{1}{2\beta}\right)\ddot{\boldsymbol{u}}_n - \frac{1}{\beta\Delta t}\dot{\boldsymbol{u}}_n + \frac{1}{\beta\Delta t^2}(\boldsymbol{u}_{n+1} - \boldsymbol{u}_n) \tag{6}$$

$$\dot{\boldsymbol{u}}_{n+1} = \left(1 - \frac{\gamma}{2\beta}\right)\Delta t\ddot{\boldsymbol{u}}_n + \left(1 - \frac{\gamma}{\beta}\right)\dot{\boldsymbol{u}}_n + \frac{\gamma}{\beta\Delta t}(\boldsymbol{u}_{n+1} - \boldsymbol{u}_n) \tag{7}$$

The equation of motion (2) is written at a time instance between the time steps t and $t + \Delta t$:

$$\boldsymbol{M}\ddot{\boldsymbol{u}}_{n+1} + (1+\alpha)\boldsymbol{C}\dot{\boldsymbol{u}}_{n+1} - \alpha\boldsymbol{C}\dot{\boldsymbol{u}}_n + (1+\alpha)\boldsymbol{K}\boldsymbol{u}_{n+1} - \alpha\boldsymbol{K}\boldsymbol{u}_n = (1+\alpha)\boldsymbol{P}_{n+1} - \alpha\boldsymbol{P}_n \tag{8}$$

Substituting Eqs. (6) and (7) in Eq. (8) and moving the $t + \Delta t$ terms at the left-hand side and the t terms at the right-hand side, the final form of the linear system of equations is obtained:

$$\begin{aligned} \left(\frac{1}{\beta\Delta t^2}\boldsymbol{M} + \frac{(1+a)\gamma}{\beta\Delta t}\boldsymbol{C} + (1+a)\boldsymbol{K}\right)\boldsymbol{u}_{n+1} &= (1+\alpha)\boldsymbol{P}_{n+1} - \alpha\boldsymbol{P}_n + \alpha\boldsymbol{K}\boldsymbol{u}_n \\ &+ \boldsymbol{C}\left(\frac{(1+\alpha)\gamma}{\beta\Delta t}\boldsymbol{u}_n + \left(\frac{\alpha\gamma}{\beta} + \frac{\gamma}{\beta} - 1\right)\dot{\boldsymbol{u}}_n \right. \\ &\left. + (1+\alpha)\left(\frac{\gamma}{2\beta} - 1\right)\Delta t\ddot{\boldsymbol{u}}_n\right) \\ &+ \boldsymbol{M}\left(\frac{1}{\beta\Delta t^2}\boldsymbol{u}_n + \frac{1}{\beta\Delta t}\dot{\boldsymbol{u}}_n + \left(\frac{1}{2\beta} - 1\right)\ddot{\boldsymbol{u}}_n\right) \end{aligned} \tag{9}$$

which can be written in the more compact form:

$$\boldsymbol{K}_{eff}\boldsymbol{u}_{n+1} = \boldsymbol{P}_{eff(n+1)} \tag{10}$$

This equation can be solved for the unknown displacement vector $\boldsymbol{u}_{n+1}$ at time $t + \Delta t$.

In this study, the shear wave propagates through an underlying bedrock layer considered homogeneous to the soil layer above. The compliance of the underlying bedrock layer is taken into account by the addition of viscous dampers attached to the base nodes of the soil model [24]. Both 1D and 2D shear wave propagation is considered, as illustrated in Figs. 1 and 2 respectively. For the 1D shear wave propagation, the shear stress is given by:

$$\tau_{xy} = G\frac{\partial u}{\partial y} \tag{11}$$

where u is the displacement along the horizontal axis x. The nodal forces of a unit area 1D element of height h_{el} illustrated in Fig. 1 are calculated as follows:

$$\begin{Bmatrix} F_i \\ F_j \end{Bmatrix} = \frac{1}{h_{el}}\begin{bmatrix} G & -G \\ -G & G \end{bmatrix}\begin{Bmatrix} u_i \\ u_j \end{Bmatrix} \tag{12}$$

and the matrix on the right-hand side of Eq. (12) corresponds to the stiffness matrix for the 1D case. For the 2-dimensional simulation of