Soil Dynamics and Earthquake Engineering 92 (2017) 176–182

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

A fast and accurate method to compute dispersion spectra for layered media using a modified Kausel-Roësset stiffness matrix approach

Jyant Kumar*, Tarun Naskar

Department of Civil Engineering, Indian Institute of Science, Bangalore 560012, India

ARTICLE INFO

Keywords:
Dispersion
Non-destructive testing
Shear wave velocity
Surface wave tests
Wave propagation

ABSTRACT

This brief article presents a simple modification to the widely-used Kausel-Roësset *Stiffness Matrix Method* (SMM), and in particular to its implementation in the context of the *Thin-Layer Method* (TLM). This modification allows making fast and accurate computations of wavenumber spectra even for layered media underlain by infinitely deep half-spaces. As is well known, the TLM uses a finite element expansion in the depth direction, which in principle disallows exact representations of infinitely deep media other than through Paraxial Approximations or Perfectly Matched Layers. However, with the modification presented herein, that obstacle is removed. The very simple method is first presented and then demonstrated by means of examples involving layered half-spaces.

1. Introduction

A dispersion spectrum is a plot showing phase velocities in terms of frequencies for the various wave modes that may propagate in a waveguide defined by a horizontally layered medium. Such spectra are widely used for miscellaneous purposes in seismology, seismic engineering and soil dynamics, among other fields. One of the most common applications is the seismic inversion of vertically inhomogeneous soils, that is, in the measurement and prediction of the material properties of the underlying soil as a function of depth on the basis of the seismic signatures observed at the surface elicited by miscellaneous sources. A very widely used and well-known method for that purpose is the SASW method (Spectral Analysis of Surface Waves Method), [1–6], which consists in trying to match iteratively the wave spectra determined experimentally with the theoretical spectra obtained by numerical means, assuming for the latter case as a certain set of initial material properties. Least squares approaches are then used to iterate towards the final solution by means of the so-called forward analysis, which consists in determining numerically the wave spectra for known material properties. The forward problem is most commonly carried out by means of the Kausel-Roësset *Stiffness Matrix Method* (SMM) [7,8] in either its exact, continuous formulation, or perhaps even more commonly, with its discrete counterpart, namely the *Thin-layer Method* (TLM) [7,9,10]. This article focuses solely on the forward problem, and specifically, on a very efficient determination of the wavenumber spectra for any layered soil, including one that is underlain by an elastic half-space. It will be shown that by a simple change in variable, one can employ the TLM even when an elastic half-space is present, and therefore, that the wave spectra for such a medium can be determined efficiently and with moderate computational effort. The writers believe that in due time the presented strategy may constitute the preferred method for the computation of wave spectra, at least for media without material damping, due to its great accuracy and simplicity.

2. Stiffness matrix method

The general problem of dynamic sources acting on or within a horizontally layered medium can be solved numerically with a number of techniques, among which a most widely used method now is the *Stiffness Matrix Method* of Kausel and Roësset [7,8]. In a nutshell, when both the wave equation and the sources are cast in the frequency-wavenumber domain by means of a multi-dimensional Fourier transform in time and in the horizontal coordinate, one is ultimately led to a seemingly simple relationship of the form

$$\mathbf{p} = \mathbf{K}\mathbf{u} \tag{1}$$

where $\mathbf{p}=\mathbf{p}(\omega,k)$ and $\mathbf{u}=\mathbf{u}(\omega,k)$ are the load and displacement vectors for the layer interfaces cast in the frequency-wavenumber domain, and $\mathbf{K}=\mathbf{K}(\omega,k)$ is the stiffness or impedance matrix of the set of layers. The impedance matrix is block-tridiagonal and symmetric, and is obtained by overlapping the exact layer stiffness matrices for each layer $\mathbf{K}=\{\mathbf{K}_l\}$ for the layers 1,2,.., l.. N, including the half-space, if any. In a plane-strain formulation, the exact, individual layer stiffness matrices $\mathbf{K}_l$

* Corresponding author.
E-mail addresses: jkumar@civil.iisc.ernet.in (J. Kumar), tarun.naskar.be@gmail.com (T. Naskar).

http://dx.doi.org/10.1016/j.soildyn.2016.09.042
Received 24 May 2015; Received in revised form 21 July 2016; Accepted 25 September 2016

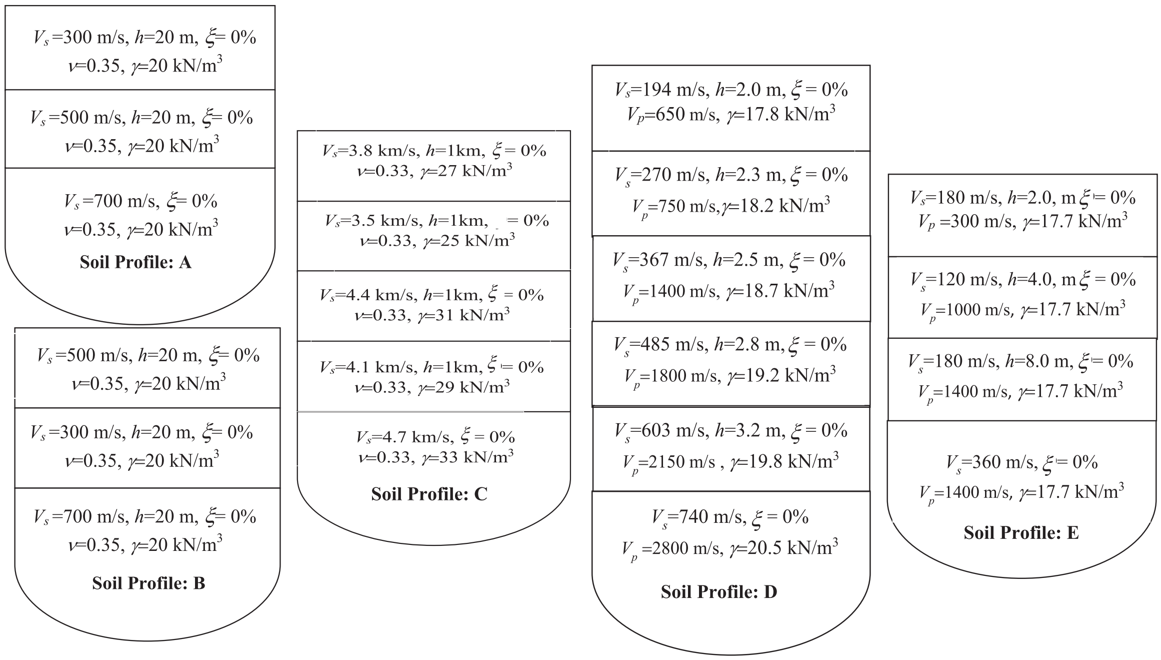

Fig. 1. Different layers and material parameters for five chosen ground profiles.

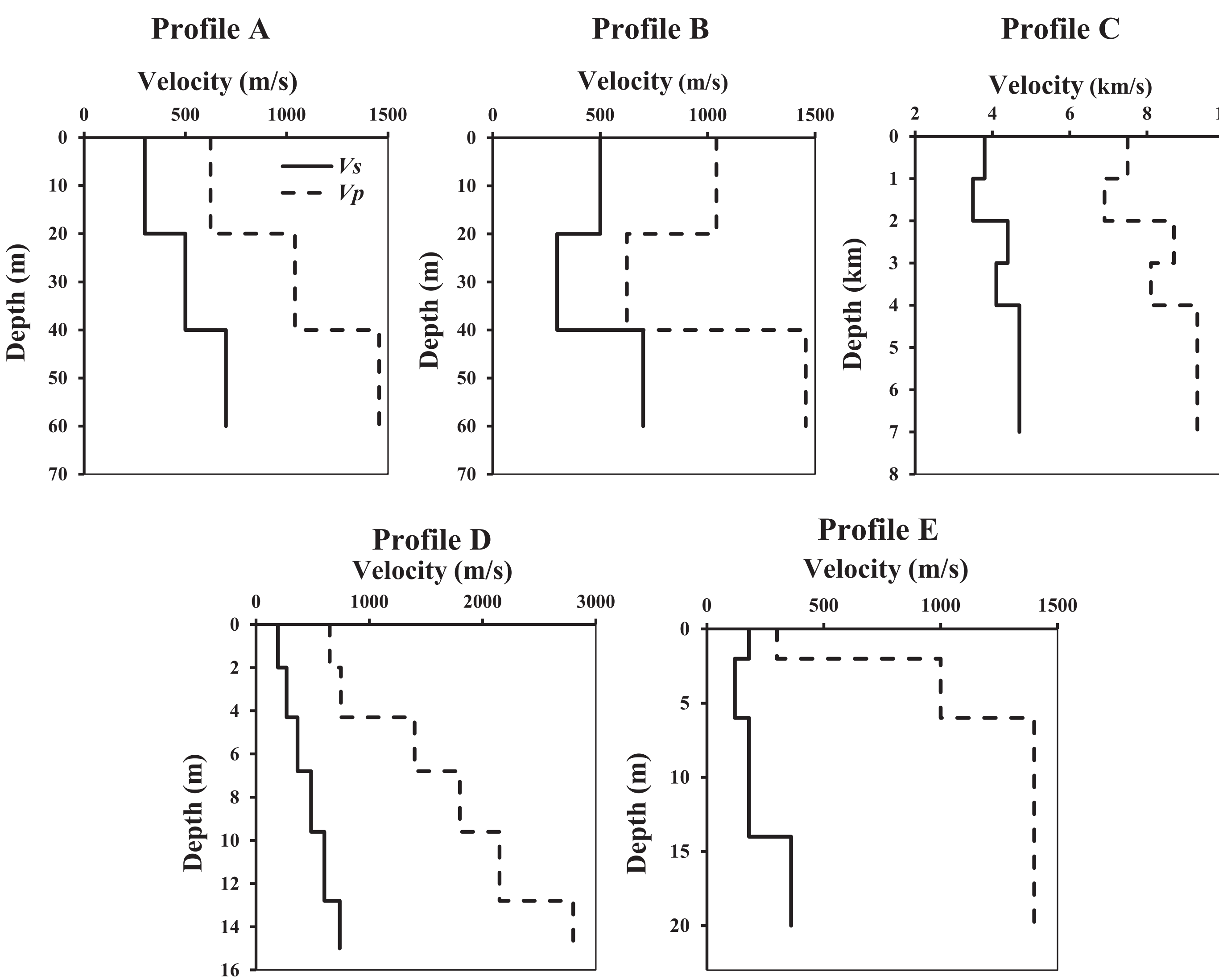

Fig. 2. The variation of V_s and V_p with depth for different chosen ground profiles.