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Vertical vibration of a massless flexible strip footing bonded to a transversely isotropic multilayered half-plane

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ABSTRACT

A rigorous analytical method is developed to analyze the vertical vibration of a massless flexible strip footing bonded to a transversely isotropic multilayered half-plane. The analytical layer-element solution for a transversely isotropic multilayered half-plane in the Fourier transform domain is first introduced for the later derivation. A pair of dual integral equations of contact stress and deflection is derived by virtue of the preceding solution and the mixed boundary conditions. By means of the classic plate theory and Jacobi orthogonal polynomials, the dual integral equations are further converted to a system of linear equations. Comparisons with existed solutions confirm the accuracy of the proposed method. More examples are given to illustrate the influence of relative rigidity ratio, transversely isotropy, double-layered characters and stratification on the vertical impedance and the contact stress.

1. Introduction

The contact problem between a plate and an elastic isotropic or orthotropic medium plays an important role in the study of soil-structure interaction. Its dynamic response is also significant to the study of seismology, earthquake engineering, machine vibrations and dynamic hardness testing, so a considerate amount of work has been done. Bycroft [1] and Robertson [2] studied the vertical dynamic response of a rigid disc resting on an elastic half-space. Later, Lysmer and Kuhlemeyer [3] investigated the displacement function of a rigid circular disc resting on or partially embedded in an elastic half-space. The steady motion of a rigid strip bonded to an elastic half-space could be found in Oien [4]. Luco and Westmann [5,6] analyzed the dynamic response of a rigid circular footing and a rigid strip footing bonded to a half-space, respectively. Soon afterwards, Luco [7] gave the impedance function for a rigid plate on a layered elastic medium. Besides, Hryniewicz [8] presented the dynamic contact stress distribution of a rigid strip on an elastic half-space. Pak and Gobert [9] considered the vertical vibration of a rigid disc with arbitrary embedment in an isotropic half-space.

However, the assumption of the plate being rigid is not always valid and the rigidity of the plate cannot be neglected in real cases, so the vibration of flexible plates has been widely discussed. Oien [10] studied the time-harmonic response of a flexible strip foundation on an elastic half space. The dynamic response of flexible rectangular foundation on an elastic half-space was carried out by Iguchi and Luco [11]. Schmidt and Krenk [12] investigated the vibration of an elastic circular plate with an elastic half-space by integral equation method with a trigonometric expansion. A series of studies related to vibrations of flexible strips on a half-plane have been presented by Karabalis and Beskos [13], Spyrakos and Beskos [14], Gaitanaros and Karabalis [15] by a hybrid BEM-FEM technique. Later, Spyrakos and Xu [16] further studied the vertical vibration of flexible strips embedded in layered elastic medium by the hybrid BEM-FEM. Riggs and Waas [17] studied the influence of circular plate's flexibility on a layered stratum. Gucunski and Peek [18,19], Gucunski [20] solved the dynamic response of a circular flexible foundation on layered soil media. Bu [21] put forward the impedance function of square foundations embedded in an incompressible half-space. Mukherjee [22] studied forced vertical vibration of an elastic elliptic plate on an elastic half-space using orthogonal polynomials. Chen and Hou [23,24] used modal analysis to evaluate a circular flexible foundation under vertical, horizontal and rocking vibration.

The papers mentioned above mainly focus on the isotropic elastic medium, however, the effect of the soils' anisotropy should be taken into consideration to simulate the real situation. Soils in geotechnical engineering generally take on transversely isotropy and stratification due to long-term sedimentation processes. As for the soil-structure interaction problems, Gazatas [25] presented a semi-analytical formulation to study the dynamic response of rigid strip footing supported

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on the surface of layered cross-anisotropic soils. Kirkner [26] developed an analytical solution for the forced vibration of a rigid surface disc on a constrained transversely isotropic elastic half-space. Eskandri-Ghadi et al. [27,28] investigated the vertical vibration of a rigid circular disc attached to and buried in a transversely isotropic half-space by Green's function method. Later, Eskandri-Ghadi et al. [29] extended their solutions into a multilayered half-space. Lin et al. [30] proposed the precise integration method for the dynamic stiffness matrices of a rigid strip footing resting on an arbitrary anisotropic layered stratum.

Recently, Ai and Liu [31] studied the axisymmetric vibration of an elastic circular plate bonded on a transversely isotropic multilayered half-space. However, the study on the dynamic response of a flexible plate on a transversely isotropic multilayered half-plane is still limited, so this paper focuses on the vertical vibration of a massless flexible strip footing bonded to a transversely isotropic multilayered half-plane for further study. In this paper, the analytical layer-element solution for the multilayered half-plane [32] is first introduced. By the application of the preceding solution and the mixed boundary conditions, a pair of dual integral equations of the contact stress and deflection is derived. By means of the classic plate theory and Jacobi orthogonal polynomials, the dual integral equations are further converted to a system of linear equations. Selected numerical results are performed to demonstrate the accuracy of present method, and to discuss the influence of relative rigidity ratio, material anisotropy and stratification. This paper provides mathematically rigorous results and offers a better understanding of the essence of the studied problem.

2. The analytical layer-element solution for a multilayered half-plane

In a Cartesian coordinate system, defined that the z-axis is normal to the plane of isotropy, an n-layered transversely isotropic elastic soil system with an underlying half-plane is illustrated in Fig. 1. The thickness of the ith layer is $h_i = H_i - H_{i-1}$, where H_i and H_{i-1} are the depths from the surface to the bottom and top of the ith layer, respectively. E_{vi}, E_{hi} and G_{vi} are the vertical Young's modulus, horizontal Young's modulus and shear modulus of the ith layer, respectively. μ_{vhi} and μ_{hi} are the Poisson's ratios characterizing horizontal strain due to stress acting parallelly and normally to the plane, respectively. In addition, ρ_i denotes the density of the ith layer. An arbitrary time-harmonic distributed load $p(x, H_i)e^{i\omega t}$ of width $2b$ is applied at the depth of H_i, where ω is the circular frequency and t is the time variable.

The analytical layer-element solution for the multilayered half-plane has been derived by Ai and Zhang [32], in which detailed definitions for the symbols used in the following derivation can be found. Let σ_z represents the normal stress components in the z direction; τ_{xz} stands for the shear stress component in the planes xz; u_x and u_z are the displacement components in the x and z directions, respectively. Stresses and displacements in this paper are time-harmonic, so the stress and displacement components can be expressed in the form of $u_x(x, z, t) = u_x(x, z)e^{i\omega t}$, $u_z(x, z, t) = u_z(x, z)e^{i\omega t}$, $\sigma_z(x, z, t) = \sigma_z(x, z)e^{i\omega t}$, and $\tau_{xz}(x, z, t) = \tau_{xz}(x, z)e^{i\omega t}$. For brevity, the harmonic time factor $e^{i\omega t}$ is suppressed from all expressions in this paper.

The whole problem is discussed in the Fourier transform domain. According to Sneddon [33], the Fourier transform with respect to the variable x and its inversion are defined as:

$$(\bar{u}_x, \bar{u}_z, \bar{\sigma}_z, \bar{\tau}_{xz}) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}(iu_x, u_z, \sigma_z, i\tau_{xz})e^{-i\xi x}dx \tag{1a}$$

$$(u_x, u_z, \sigma_z, \tau_{xz}) = \int_{-\infty}^{+\infty}(-i\bar{u}_x, \bar{u}_z, \bar{\sigma}_z, -i\bar{\tau}_{xz})e^{i\xi x}d\xi \tag{1b}$$

where ξ is the Fourier transform parameter with respect to the variable x, and $i = \sqrt{-1}$.

In order to simplify the analysis, two variables are defined as follows:

$$\bar{\mathbf{U}}(\xi, z) = [\bar{u}_x(\xi, z), \bar{u}_z(\xi, z)]^{T} \tag{2a}$$

$$\bar{\mathbf{V}}(\xi, z) = [\bar{\tau}_{xz}(\xi, z), \bar{\sigma}_z(\xi, z)]^{T} \tag{2b}$$

The solutions for a single layer with a finite thickness and a half-plane are given in the following matrix form:

$$\begin{bmatrix} -\bar{\mathbf{V}}(\xi, 0) \\ \bar{\mathbf{V}}(\xi, z) \end{bmatrix} = \mathbf{K}(\xi, z)\begin{bmatrix} \bar{\mathbf{U}}(\xi, 0) \\ \bar{\mathbf{U}}(\xi, z) \end{bmatrix} \tag{3a}$$

$$[-\bar{\mathbf{V}}(\xi, z)] = \mathbf{K}_h(\xi, z)[\bar{\mathbf{U}}(\xi, z)] \tag{3b}$$

where $\mathbf{K}(\xi, z)$ and $\mathbf{K}_h(\xi, z)$ are symmetric matrices of order 4×4 and 2×2. The analytical layer-element associates the displacements and stresses of $z = 0$ and arbitrary depth z in the Fourier transformed domain. The critical analytical details of the explicit forms of the layer-element are given in Ref. [32].

By applying Eq. (3a) to each finite layer and Eq. (3b) to the underlying half-plane, the global stiffness matrix of the multilayered half-plane in the Fourier transform domain can be assembled in the form of:

$$\begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \vdots \\ -\bar{\mathbf{F}}(\xi, H_i) \\ \vdots \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} = \begin{pmatrix} \mathbf{K}_1 & & & & \\ & \mathbf{K}_2 & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & \mathbf{K}_n \\ & & & & & \mathbf{K}_h \end{pmatrix}_{(2n+2)\times(2n+2)} \begin{pmatrix} \bar{\mathbf{U}}(\xi, H_0) \\ \bar{\mathbf{U}}(\xi, H_1) \\ \vdots \\ \bar{\mathbf{U}}(\xi, H_i) \\ \vdots \\ \bar{\mathbf{U}}(\xi, H_{n-1}) \\ \bar{\mathbf{U}}(\xi, H_n) \end{pmatrix}$$

where $[\bar{\mathbf{U}}(\xi, H_0), \bar{\mathbf{U}}(\xi, H_1), \ldots, \bar{\mathbf{U}}(\xi, H_i), \ldots, \bar{\mathbf{U}}(\xi, H_{n-1}), \bar{\mathbf{U}}(\xi, H_n)]^{T}$ is the unknown displacement vector in the Fourier transformed domain; $\mathbf{K}_i = \mathbf{K}(\xi, h_i)$ and $\mathbf{K}_h$ represent the analytical layer-element of the ith