Soil Dynamics and Earthquake Engineering 90 (2016) 494–509

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

Transmitting boundary for transient analysis of wave propagation in layered media formulated based on acceleration unit-impulse response

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A R T I C L E I N F O

Keywords:
Scaled boundary finite element method (SBFEM)
Acceleration unit-impulse response matrix
Transmitting boundary
Layered soil
Dynamic soil-structure interaction

A B S T R A C T

An efficient transmitting boundary (TB) is developed for transient analysis of scalar wave as well as vector wave motion in layered media of constant depth, it is suited for both waves propagating out to infinity and for evanescent waves. The TB is formulated based on the acceleration unit-impulse response, such that singular term does not present, which results in simplifying the solution. The scaled boundary finite element method (SBFEM) is employed for the derivation. For numerical implementation, the very efficient scheme proposed by Radmanović and Katz is applied for the solution of the acceleration unit-impulse response matrix and the evaluation of interaction forces, significant reduction of computational effort can be achieved. In comparison with the various schemes of TBs available in the literature, the proposed approach has following advantages: it is accurate, the computational cost is less and stability of the solution can be ensured. The extension to the case of orthogonal anisotropic media can be performed without extra effort.

1. Introduction

Wave propagation in unbounded media is of great importance in many fields of application, such as acoustics, geophysics, meteorology, oceanography, elastodynamics and electromagnetics etc. A common approach for numerically modeling wave propagation in unbounded media is to limit the computational domain of interest via suitable transmitting boundaries imposed at the truncated surfaces. Apart from transmitting boundary (TB), in the literature some other names, such as non-reflecting boundary, absorbing boundary, open boundary, artificial boundary etc. were used. TB is designed to represent the effect of residual infinite portion of the domain on wave propagation. It takes account of the radiation damping in infinity and it should satisfy the condition that no (or little) spurious wave reflection occurs. Its design has also to consider the question of stability, accuracy, efficiency of the numerical analysis and easy to implement.

Over the past decades since 1980s significant progress has been made concerning the development of numerical modeling of TB. This is reflected in a number of review articles [1–5]. In the following, only a few significant ones are presented for reference.

Most of the currently used techniques for setting TBs can basically be classified into two groups. The methods from the first group constitute the global TB, which is global in space and time. The radiation condition is satisfied rigorously, and thus leads to high accuracy and robustness of the numerical procedure, but it appears to be computationally expensive. The methods from the second group are local TB, i.e. local in space and time. It is designed aiming to be algorithmically simple and numerically inexpensive. However, it usually lacks accuracy of computations.

Early local TBs, such as the most popular viscous boundary [6], are low-order TB, where only first-order approximation can be reached, their accuracy is low. Only since the mid-1990s, practical high-order TBs have been devised [5], with rising the order of TBs, accuracy increases. Despite being of an arbitrary high-order, these TBs do not involve high derivatives owing to the use of specially defined auxiliary variables. Nevertheless, with increasing the 'order' for better accuracy, more additional degrees of freedom (DOFs) are needed to construct the local TB scheme. Recently, some new approaches for constructing time-domain local TBs are of interest. Bazyar and Song [7] proposed a high-order TB based on continued-fraction solution of the scaled boundary finite element (SBFE) wave equation and Birk et al. [8] proposed the improved version of it. This TB is suitable to deal with scalar waves as well as vector waves (applying to both isotropic and anisotropic medium). However, a high-order TB is not necessary exact. Most of the existing high-order TBs are singly asymptotic at the high-frequency limit, they are suited for waves propagating out to infinity and are not suited for evanescent waves. To circumvent such difficulty, Prempramote et al. [9] proposed a high-order doubly asymptotic TB by

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http://dx.doi.org/10.1016/j.soildyn.2016.09.021
Received 21 January 2016; Received in revised form 12 September 2016; Accepted 16 September 2016

extending the work of [7] to include both high- and low-frequency expansions. It successfully deals with the evanescent waves, unfortunately, it is limited to scalar waves only.

As far as the family of global TBs is concerned. The boundary-element method (BEM) and scaled boundary finite element method (SBFEM) are popular. BEM [10,11] has distinct benefits for modeling wave propagation in unbounded domain that the radiation condition at infinity is satisfied rigorously and only the boundary of the domain needs to be discretized, the spatial dimension of the problem is reduced by one. SBFEM [12,13] shares the above important features with the BEM, but no fundamental solution is required. Practical implementation of the global TBs for time-dependent problems generally presents more substantial difficulties, because nonlocality of the TB appears not only in space, but also in time. Much research has been devoted to reduce the computational effort for the evaluation of the convolution integrals and the soil-structure interaction forces of the unbounded medium. Paronesso and Wolf [14,15] proposed a rational function of Legendre polynomials to approximate the unit-impulse response and to ease the evaluation of interaction forces in the time domain. Zhang et al. [16] make approximations both in time and space: (1) the acceleration unit-impulse response matrix is approximated by a few linear segments, such that larger time-steps can be used. (2) The soil-structure interface is partitioned into several parts, the unit-impulse response matrices are calculated separately for each part and then assembled, which makes the global matrix being sparse. In the approach of Yan et al. [17], the unit-impulse response matrix is diagonized using the procedure described in [14,18] to simplify the solution of convolution integrals. The work of Radmanović and Katz [19] is most notable, which brings two essential improvements for the solution of acceleration unit-impulse response matrix M(t) and the evaluation of soil-structure interaction force R(t): (1) A piece-wise linear change of M(t) is assumed in combination with an extrapolation parameter θ to ensure stability of the computation, furthermore, after M(t) is linearized at late times $t > t_n$, M(t) is approximated using extrapolation from the latest time segment. Such linearization has also been exploited in [16] and [17]. (2) An efficient scheme based on integration by parts is used for the evaluation of the convolution integral for R(t). These two enhancements lead to a very significant reduction of computational effort. In case of global TB for layered soil strata, BEM has some difficulties to deal with, because all the interfaces between layers have to be discretized. Chen et al. [20,21] presented a global TB for transient analysis of layered strata using displacement unit-impulse response matrix based on SBFEM, where the frequency dependent dynamic stiffness $[S(\omega)]$ is decomposed into a singular-part $[S_s(\omega)]$ and a remaining regular-part $[S_r(\omega)]$. The interaction force is evaluated separately for singular part and regular part respectively.

In this paper, a TB for transient analysis of wave propagation in layered soil strata based on acceleration unit-impulse response is presented. It is global in space and in time. Numerical examples show that it applies to both out-of-plane motion of scalar waves and in-plane motion pf vector waves; it is suited for waves propagating out to infinity and also for evanescent waves. In addition, numerical examples also validate that the very efficient scheme presented by Radmanović and Katz for half-space domains for the solution of unit-impulse response matrix and the evaluation of convolution integrals is equally effective to deal with the layered strata. The assumption of piece-wise linear change of M(t) allows larger time-step to be used; The introduction of an extrapolation parameter θ ensures stability of computation; and the scheme based on integration by parts simplifies the solution of convolution integrals. As a result, significant reduction of computational effort can be achieved. Taking into consideration the fact that additional DOFs are needed for the high-order local TB, the higher the order, the more additional DOFs are required to eliminate the high derivatives from the high-order TB [5], while in the proposed TB no additional DOFs is required. In this regard, it is expected that the computational efficiency of proposed global TB is in competition with the local high-order TB.

2. Formulation of the SBFE governing equations for layered media

Consider the plane wave motion in horizontally anisotropic layered soil strata as shown in Fig. 1. In this paper, we will address problems for isotropic or orthogonal anisotropic medium, where the wave motion equation can be decoupled into the in-plane motion of vector P-SV waves and out-of-plane motion of scalar SH wave. For each layer, the stress-strain relationship is expressed in the following form

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\end{Bmatrix}=\begin{bmatrix}D_{11} & D_{12} & D_{13}\\ D_{12} & D_{22} & D_{23}\\ D_{13} & D_{23} & D_{33}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\end{Bmatrix},\ \begin{Bmatrix}\tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}D_{44} & 0 & 0\\ 0 & D_{55} & 0\\ 0 & 0 & D_{66}\end{bmatrix}\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix} \quad (1)$$

.

For a transverse isotropic medium, the independent elastic material constants reduce to five, the following equalities hold:

$$D_{22}=D_{11},\ D_{13}=D_{23},\ D_{55}=D_{44},\ D_{66}=\frac{1}{2}(D_{11}-D_{12}) \quad (2)$$

Thermodynamic considerations require that the strain energy in an elastic material due to all possible stress fields be non-negative. This imposes certain restrictions on the acceptable range of elastic constants. Carrier [22] proposed the following constraint relationship as follows, where the shear modulus D_{44} and D_{55} is expressed as a function of the other elastic constants.

$$D_{44}=D_{55}=\frac{D_{11}D_{33}-D_{13}^2}{D_{11}+2D_{13}+D_{33}} \quad (3)$$

And for an isotropic medium, we have Poisson's ratio $\nu=\nu_{HH}=\nu_{HV}$and Young's modulus of elasticity $E=E_{HH}=E_{HV}$(subscripts H and V denote horizontal and vertical components respectively), which leads to $D_{11}=D_{13}=\lambda+2G$, $D_{12}=D_{13}=\lambda$ and $D_{44}=D_{66}=G$ with λ and G denoting the Lame's constants, the

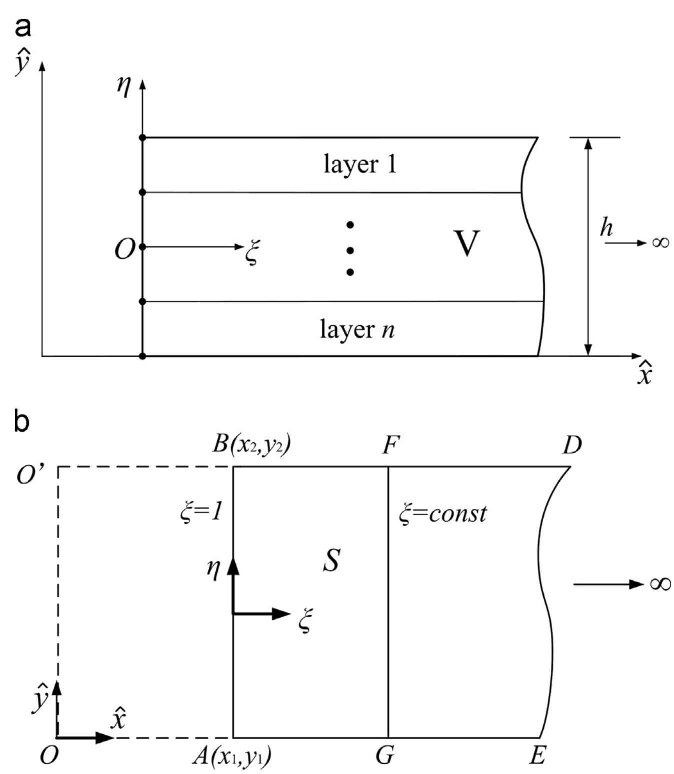

Fig. 1. 2D layered medium of constant depth. (a) Coordinate transformation of the soil strata. (b) Sublayer element.