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Dynamic reliability analysis of slopes based on the probability density evolution method

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ABSTRACT

Earthquake ground motions display random behavior; therefore, it is necessary to investigate the seismic response of engineering structures using stochastic analysis methods. The probability density evolution method (PDEM) is applied in stochastic seismic response analysis and dynamic reliability evaluation of slope stability. Combining PDEM with finite-element dynamic time-series analysis, this study analyses the stochastic seismic response of a slope under random earthquake ground motion. Comparison between the results of the Monte Carlo stochastic simulation and PDEM analysis demonstrates the effectiveness and high precision of the PDEM method. We assess the stability of the slope under earthquake conditions using the safety factor criterion to obtain the stability probability of the slope. Compared with traditional equilibrium methods, the reliability analysis can directly reflect the failure probability and degree of safety of the slope, demonstrating a novel approach to slope stability assessment using a random dynamic method.

1. Introduction

In the past few decades, the random reliability theory has been used to address geotechnical engineering problems. Using the Newmark model, Mankelow and Murphy [1] obtained the mean and standard deviation of the Newmark displacement with a Gaussian distribution. To investigate the spatial variation of soil properties, the random finite element method (FEM) was incorporated into slope stability analysis as a probabilistic analysis tool [2,3]. Based on the low-discrepancy sequence Monte Carlo (MC) method, Shinoda et al. [4] calculated the limit state exceedance probability of typical earth dams and geosynthetic-reinforced soil slopes under earthquake loading. For slopes under seismic loading, several stochastic analysis approaches were developed for stability analysis and slope reliability evaluation. Different models were developed for seismic reliability assessment of earth slopes with short term stability [5]. Peng et al. [6] proposed a neural network method to consider the reliability of earth slopes. Using stochastic parameters such as the internal friction angle, cohesion, and soil unit weight, the MC method was applied to analyze an infinite slope subjected to pseudo-static earthquake loading [7].

Most of the studies on slope stability based on reliability methods only investigated the probability of earth slope failure triggered by earthquakes based on the Newmark model and static or pseudo-static analysis. However, there are two aspects that limit the effectiveness of these methods [8]. First, most of the above methods do not take into account the characteristics of the seismic ground motion and the seismic wave amplification by the soil mass. Second, most of the methods do not account for the dynamic nonlinear behavior of soil under the seismic excitation. Therefore, it is necessary to investigate the seismic stability of slopes from the viewpoint of stochastic vibration using dynamic time-history analysis.

The traditional MC method [9,10] significantly improves our understanding of stochastic vibration analysis in the field of slope engineering; however, engineers are reluctant to adopt it in seismic assessments of slope stability because of certain limitations [11]. Therefore, in this study we analyze the slope stability under stochastic earthquake excitation based on the probability density evolution method (PDEM) [12,13]. By applying deterministic dynamic time-history analyses of seismic excitation using FEM combined with the PDEM, we obtain the probability density functions of the seismic responses of the slope under stochastic earthquake ground motion, and calculate the seismic dynamic reliability precisely.

This paper proposes an efficient method for dynamic analysis of slopes subjected to recently developed PDEM. It is completely different from the traditional stochastic reliability analysis method, the PDEM can combine the stochastic dynamic analysis and currently various advanced deterministic slope stability analysis methods. According to the PDEM, the stochastic seismic response analysis of slope is

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translated into serials of deterministic responses analyses. The time history dynamic method is used for the deterministic seismic calculation. It can reflect the characteristics of the seismic ground motion and the dynamic nonlinear behavior of soil under the seismic excitation are used in this paper for the seismic dynamic analysis of slope under the earthquake loading. The study addresses a new stochastic seismic dynamic method for the slope seismic stability analysis; the PDEM has been shown to be more than 14 times more efficient than traditional MC method. And this paper is the first succeeded in investigating the stability reliability of slope based on seismic dynamic time history.

2. The PDEM equation and dynamic reliability

Without loss of generality, the dynamic balance equation of the soil slope subjected to earthquake motion can be expressed as follows [12]:

$$M\ddot{U} + C\dot{U} + \boldsymbol{f}(U, \dot{U}) = -\boldsymbol{MI}\ddot{x}_g(\boldsymbol{\Theta}, t);\ \dot{U}(t_0) = \dot{x}_0,\ \boldsymbol{U}(t_0) = x_0, \tag{1}$$

where $\boldsymbol{M}$ and $\boldsymbol{C}$ are the mass and damping matrices, respectively and $\boldsymbol{f}$ is the nonlinear restoring force vector; $\ddot{\boldsymbol{U}}$, $\dot{\boldsymbol{U}}$, and $\boldsymbol{U}$ are the acceleration, velocity, and displacement vectors, respectively; $\boldsymbol{I}$ is the unit vector, $\ddot{x}_g$ is the earthquake ground motion process, and $\boldsymbol{\Theta}$ is a random vector.

The seismic response vector $\boldsymbol{H}$ is composed of the physical quantities which attract us being evaluated and can be described as

$$\boldsymbol{H} = (H_1, H_2, \ldots, H_m)^T. \tag{2}$$

According to the probability conservation principle, the stochastic system is conservative and composed of $(\boldsymbol{H}(t), \boldsymbol{\Theta})$. Therefore, its joint probability $\boldsymbol{p_{H\Theta}}(h, \boldsymbol{\theta}, t)$ satisfies the generalized probability density evolution equation [12,14]

$$\frac{\partial p_{H\Theta}(h,\boldsymbol{\theta},t)}{\partial t} + \sum_{j=1}^{m} \dot{H}_j(\boldsymbol{\theta}, t)\frac{\partial p_{H\Theta}(h,\boldsymbol{\theta},t)}{\partial h_j} = 0. \tag{3}$$

The initial condition of Eq. (3) is

$$\boldsymbol{p_{H\Theta}}(h, \boldsymbol{\theta}, t_0) = p_{\Theta}(\boldsymbol{\theta}, t)\delta(h - h_0), \tag{4}$$

where h_0 is the initial value of $\boldsymbol{H}(t)$ and $\delta(\cdot)$ is the Dirac function. The probability density function $p_H(H, t)$ of $\boldsymbol{H}(t)$ is given by

$$p_H(h, t) = \int_{\Omega\Theta} \boldsymbol{p_{H\Theta}}(h, \boldsymbol{\theta}, t)\mathrm{d}\boldsymbol{\theta}. \tag{5}$$

In Eq. (3), the dimensions of the PDEM equation depend on the dimensions of the physical parameters being evaluated, and are unrelated to the dimensions of the primitive stochastic dynamic system. However, the dimensions of the classical density evolution equations cannot be smaller than those of the primitive stochastic dynamic system.

Any physical parameters which attracted us (e.g. displacement and safety factor) can be chosen as the random variable in PDEM equation. For the slope stability analysis in this study, the selected physical parameter is the time series of the safety factor. The one-dimensional PDEM equation for the slope analysis can then be written as [10,14]

$$\frac{\partial p_{F_s\Theta}(F_s,\boldsymbol{\theta},t)}{\partial t} + \dot{F}_s(\theta, t)\frac{\partial p_{F_s\Theta}(F_s,\boldsymbol{\theta},t)}{\partial F_s} = 0, \tag{6}$$

where F_s is the dynamic seismic time series of the safety factor of the slope, which can be calculated by the formulas presented in the following section.

Eq. (3) can be solved by the following numerical method [10].

(1) Select the representative discretized points $\boldsymbol{\theta_q}(q = 1, 2, \ldots, n_{pt})$ in the basic random variable space $\boldsymbol{\Theta}$ and determine the corresponding probability. In this paper, the random source is the earthquake ground motion; therefore, the basic random variables relate to the earthquake ground motion parameters. In this paper, the randomness of the earthquake ground motion mainly embodies in peak acceleration value (PGA) and the frequency spectrum distribution. The randomness is characterized by the basic random variables; meanwhile, the stochastic earthquake ground motion can reflect uncertainties of intensity and frequency. With the bi-modulation function of intensity and frequency, the model also can respond the non-stationary characteristic of intensity and frequency.
(2) For the determined $\boldsymbol{\theta_q}$, solve the dynamic equation (Eq. (1)) with the given earthquake excitations, and obtain the velocity of the seismic response.
(3) Substitute the velocity into the PDEM equation and solve it by the finite difference method.
(4) Calculate the sum of the results for $q = 1, 2, \ldots, n_{pt}$, and obtain the required probability density function.

The effectiveness and accuracy of this method for solving geotechnical engineering problems were verified by Huang et al. [10,13,14].

Finally, combining the data of an equivalent extreme event [14] with the PDEM solution, the seismic dynamic reliability can be obtained. For the PDEM, the extreme value of the each response of deterministic time history analysis makes up a virtual stochastic process, and which random process can be regarded as the input for the PDEM, therefore, solving the PDEM equation, the dynamic reliability is obtained. Here, should be emphasized that PDEM is a newly developed stochastic dynamic analysis method, it can be based on the existing deterministic methods (e.g. FEM, limit equilibrium method and so on) and which does not lead to the violation of objective physical law.

3. The stochastic ground motion

According to the numerical steps for solving the PDEM equation, first we need to obtain the time series of the stochastic earthquake ground motion. The acceleration time series of the stochastic seismic ground motion are generated by the spectral representation of the ground motion and the stochastic function [15]. Based on the work of Cacciola and Deodatis [16], we introduce the generalized Clough and Penzien earthquake ground motion power spectrum density function model in the form of the following time evolution power spectrum [17]:

$$S_{\ddot{X}_g}(t, \omega) = |f(t)|^2 \cdot \frac{\omega_g^4(t) + 4\xi_g^2(t)\omega_g^2(t)\omega^2}{\left[\omega^2 - \omega_g^2(t)\right]^2 + 4\xi_g^2(t)\omega_g^2(t)\omega^2} \cdot \frac{\omega^4}{\left[\omega^2 - \omega_f^2(t)\right]^2 + 4\xi_f^2(t)\omega_f^2(t)\omega^2} \cdot S_0(t), \tag{7}$$

where $f(t)$ is the intensity modulation function, represented by

$$f(t) = \left[\frac{t}{c}\exp\left(1 - \frac{t}{c}\right)\right]^d. \tag{8}$$

$c = 7\mathrm{s}$ is the arrival time of the peak ground acceleration and $d = 2$ is the parameter that controls the shape of $f(t)$ and effectively regulates the enhancement and attenuation of the stochastic earthquake process.

In the generalized Clough and Penzien power spectrum model, the following parameters reflect the non-stationary characteristics of the earthquake ground motions [15,17]

$$\omega_g(t) = \omega_0 - a\frac{t}{T} \quad \xi_g(t) = \xi_0 + b\frac{t}{T}, \tag{9}$$

$$\omega_f(t) = 0.1\omega_g(t) \quad \xi_f(t) = \xi_g(t). \tag{10}$$

In Eq. (9), the site parameters $\omega_g(t)$ and $\xi_g(t)$ are the dominant angular frequency and damping ratio, respectively, which change with time; the initial values of $\omega_g(t)$ and $\xi_g(t)$ are $\omega_0 = 11\mathrm{s}^{-1}$ and $\xi_0 = 0.85$; $a = 8\mathrm{s}^{-1}$ and $b = 0.15$ are the variance ratios of the site parameters; ω_0, ξ_0, a and b can be determined according to site classification and the earthquake characteristics; $T = 20\mathrm{s}$ is the duration of the time series. In Eq. (10), $\omega_f(t)$, and $\xi_f(t)$ are the corresponding filtering parameters. The site parameters and filtering parameters are linear functions of time; thus, $\omega_f(t)$ and $\xi_f(t)$ are parameters that vary in time within a certain range.