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# Influences of Biot's compressible parameters on dynamic response of vertically loaded multilayered poroelastic soils

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ABSTRACT

The problem is formulated on the basis of Biot's theory and the analytical layer element method, and the global stiffness matrix for the multilayered soil is established by combining continuity conditions of adjacent layers and boundary conditions based on the analytical layer element for a single poroelastic saturated layer in the Hankel transformed domain. Solutions in frequency domain are obtained by taking the Hankel inverse transform. Selected numerical examples are performed to validate the correctness of the present method and to discuss the influences of compressibility parameters of soil grain and pore fluid as well as the influence of soil stratification on vertical displacement and pore pressure.

## 1. Introduction

The dynamic behavior of poroelastic saturated soils has been the subject of intensive investigations for decades. Biot [1] developed the theory of wave propagation in saturated media within the small strain theory. Paul [2] studied the Lamb's problem in a saturated half-space. Halpern and Christiano [3] solved the problem associating with the saturated half-space due to a surface harmonic load by applying the potential functions decomposition and Hankel transform. Philippacopoulos [4,5] investigated the wave propagation and Lamb's problem in partially saturated and fluid-saturated porous media, in which the solid damping and viscosity of pore fluid were taken into consideration. Senjuntichai and Rajapakse [6] and Rajapakse and Senjuntichai [7] studied the dynamic response of a dissipative poroelastic half-plane and obtained solutions for a multilayered poroelastic medium due to time-harmonic loads, respectively. Cai et al. [8] obtained the fundamental solutions for single-layered isotropic saturated soil with a subjacent rock-stratum subjected to harmonic excitations. Lu and Hanyga [9] studied the fundamental problem of a layered porous half space due to a vertical point force and a point fluid source. However, none of them investigated the influences of Biot's parameters for compressibility of two-phased soils.

This investigation aims to present the fundamental solutions for multilayered poroelastic soils subjected to an axisymmetric harmonic excitation and to study influences of Biot's compressibility parameters and soil stratification property. The analytical layer element [10–12] for a single soil layer (as shown in Fig. 1) is established with the aid of Laplace-Hankel transform. The global stiffness matrix for the multilayered soil can be developed by combining continuity conditions of adjacent layers and boundary conditions based on the analytical layer element. Solutions for multilayered soil media in frequency domain are derived by taking the Hankel inverse transform. Selected numerical examples are performed to validate the accuracy of the present method and to discuss the influences of compressibility parameters and soil stratification.

## 2. Analytical layer element for a single poroelastic soil layer

The derivation of analytical layer element is based on the following assumptions: a) the soil medium is elastic and isotropic; b) the study is under small deformation assumption; c) the soil porosity is uniform; and d) the interfaces of adjacent soil layers are assumed to be parallel to the ground surface. The equilibrium equations of a poroelastic medium are expressed as follows [13]

$$\mu\nabla^2 u + (\lambda + \alpha^2 M + \mu)\frac{\partial e}{\partial r} - \mu\frac{u}{r^2} - \alpha M\frac{\partial \varsigma}{\partial r} = \rho\ddot{u} + \rho_f\ddot{v}_r \tag{1a}$$

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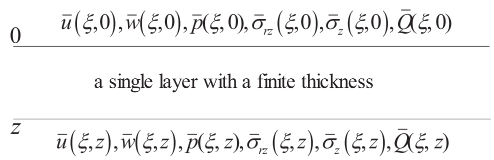


**Fig. 1.** Generalized stresses and displacements of a single layer with a finite thickness.

$$\mu\nabla^2 w + (\lambda + \alpha^2 M + \mu)\frac{\partial e}{\partial z} - \alpha M\frac{\partial \varsigma}{\partial z} = \rho\ddot{w} + \rho_f\ddot{v}_z \tag{1b}$$

where the over dot is the derivation with respect to $t$; $\mu$, $\lambda$, $\rho$ are the shear modulus, Lame's constant, density of the bulk material, respectively; $\rho_f$ is the density of pore water; $\alpha$ and $M$ are the Biot's parameters accounting for compressibility of soil grain and pore fluid; $u$, $w$, $v_r$, $v_z$ are the displacement components of soil grain and pore fluid in $r$ and $z$ directions, separately; $e = \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z}$, $\varsigma = -\left(\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z}\right)$.

The balance equations of fluid motion and the seepage continuity equation are given as follows [13]

$$\alpha M\frac{\partial e}{\partial r} - M\frac{\partial \varsigma}{\partial r} = b\dot{v}_r + \rho_f\ddot{u} + m\ddot{v}_r \tag{2a}$$

$$\alpha M\frac{\partial e}{\partial z} - M\frac{\partial \varsigma}{\partial z} = b\dot{v}_z + \rho_f\ddot{w} + m\ddot{v}_z \tag{2b}$$

$$\alpha Me - M\varsigma = -p \tag{3}$$

where $b = \eta_0/k$, $\eta_0 = n\rho_f g$, $m = \rho_f/n$; $\eta_0$, $k$, $n$ denote pore fluid viscosity, soil permeability and porosity; $p$ denotes pore pressure ($p$ is considered positive when the force acting on the fluid is a pressure).

Due to constitutive equations and principle of effective stress [10] as well as Hankel transform [14], we obtain

$$\overline{\mathbf{V}}(\xi, z) = \mathbf{H}\begin{bmatrix}\overline{\mathbf{W}}(\xi, z) \\ \frac{\mathrm{d}\overline{\mathbf{W}}(\xi, z)}{\mathrm{d}z}\end{bmatrix} \tag{4}$$

where $\mathbf{H} = \begin{bmatrix} 0 & -\mu\xi & 0 & \mu & 0 & 0 \\ \lambda\xi & 0 & -\alpha & 0 & \lambda + 2\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & -1/\delta \end{bmatrix}$, $\overline{\mathbf{V}}(\xi, z) = [\bar{\sigma}_{rz} \ \ \bar{\sigma}_z \ \ \overline{Q}]^T$, $\overline{\mathbf{W}}(\xi, z) = [\bar{u} \ \ \bar{w} \ \ \bar{p}]^{\mathrm{T}}$; $\bar{u}$, $\bar{w}$, $\bar{p}$ are the Hankel transforms of $u$, $w$, $p$; $\delta = \rho_f g/k$, $\xi$ is the Hankel transform parameter with respect to $r$; $\bar{\sigma}_{rz}$, $\bar{\sigma}_z$, $\overline{Q}$ are the Hankel transforms of $\sigma_{rz}$,$\sigma_z$, $Q$, which denote the shear stress in the plane $r$-$z$, the total normal stress of solid matrix and flow quantity through unit cross section area in $z$ direction.

The motion in this investigation is assumed to be time-harmonic. With the application of Hankel transform with respect to $r$ and Laplace transform with respect f to $z$ [14], Eqs. (1), (3) are written in matrix form as

$$\mathbf{A}\widetilde{\mathbf{W}}(\xi, s) = \mathbf{B}\overline{\mathbf{Y}} \tag{5}$$

where $\mathbf{A} = \begin{bmatrix} \mu l - b_2\xi^2 + b_1 & -\xi s b_2 & \xi\alpha a_3 l + a_2\xi \\ b_2\xi s & \mu l + b_2 s^2 + b_1 & -s\alpha a_3 l - s a_2 \\ \xi b_3 & s b_3 & 1 - a_3 l \end{bmatrix}$, $\widetilde{\mathbf{W}}(\xi, s) = [\tilde{u} \ \ \tilde{w} \ \ \tilde{p}]^{\mathrm{T}}$,
$\mathbf{B} = \begin{bmatrix} \mu s & \xi c_2 & a_3\alpha\xi s & 1 & 0 & -a_3\alpha\xi\delta \\ \mu\xi & s c_1 & -a_3\alpha s^2 - a_2 & 0 & 1 & a_3\alpha s\delta \\ 0 & b_3 & -a_3 s & 0 & 0 & a_3\delta \end{bmatrix}$, $\overline{\mathbf{Y}} = [\bar{u}(0) \ \ \bar{w}(0) \ \ \bar{p}(0) \ \ \bar{\sigma}_{rz}(0) \ \ \bar{\sigma}_z(0) \ \ \overline{Q}(0)]^{\mathrm{T}}$;
$a_1 = \lambda + \alpha^2 M + \mu$, $a_2 = c\omega^2\rho_f$, $a_3 = Mc$, $b_1 = \omega^2(\rho + \rho_f a_2)$, $b_2 = a_1 + \alpha M a_2$, $b_3 = M(\alpha + a_2)$, $c_1 = \mu + b_2$, $c_2 = \mu - b_2$, $l = s^2 - \xi^2$; $c = 1/(\mathrm{i}\omega b - m\omega^2)$, $\omega$ is the circular frequency; $\tilde{u}$, $\tilde{w}$, $\tilde{p}$ are Laplace transforms of $\bar{u}$, $\bar{w}$, $\bar{p}$; $s$ is the Laplace transform parameter with respect to $z$.

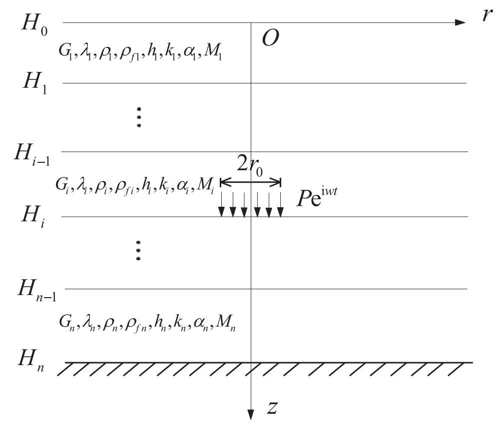


**Fig. 2.** Multilayered isotropic poroelastic soils subjected to a time-harmonic excitation.

Applying the inversion of Laplace transform with respect to s to Eq. (5), we have

$$\overline{\mathbf{W}}(\xi, z) = \mathbf{U}\overline{\mathbf{Y}} = [\mathbf{U}_1 \ \ \mathbf{U}_2]\overline{\mathbf{Y}} \tag{6}$$

where $\mathbf{U}$ is the matrix of order $3 \times 6$, which is divided into $\mathbf{U}_1$ and $\mathbf{U}_2$ of order $3 \times 3$.

According to Eq. (6), we have the following relationship in matrix form

$$\begin{bmatrix}\overline{\mathbf{W}}(\xi, 0) \\ \overline{\mathbf{W}}(\xi, z)\end{bmatrix} = \begin{bmatrix}\mathbf{I} & \mathbf{0} \\ \mathbf{U}_1 & \mathbf{U}_2\end{bmatrix}\overline{\mathbf{Y}} = \mathbf{N}\overline{\mathbf{Y}} \tag{7}$$

where $\mathbf{I}$, $\mathbf{0}$ are unit matrix and zero matrix of order $3 \times 3$, respectively.

Combining Eq. (4) and Eq. (6) leads to

$$\begin{bmatrix}-\overline{\mathbf{V}}(\xi, 0) \\ \overline{\mathbf{V}}(\xi, z)\end{bmatrix} = \begin{bmatrix}\mathbf{0} & -\mathbf{I} \\ \mathbf{T}_1 & \mathbf{T}_2\end{bmatrix}\overline{\mathbf{Y}} = \mathbf{M}\overline{\mathbf{Y}} \tag{8}$$

where $\mathbf{T}_1$ and $\mathbf{T}_2$ are matrices of order $3 \times 3$ and $\mathbf{M}$ is a matrix of order $6 \times 6$.

Combining Eqs. (7) and (8) leads to

$$\begin{bmatrix}-\overline{\mathbf{V}}(\xi, 0) \\ \overline{\mathbf{V}}(\xi, z)\end{bmatrix} = \mathbf{\Phi}\begin{bmatrix}\overline{\mathbf{W}}(\xi, 0) \\ \overline{\mathbf{W}}(\xi, z)\end{bmatrix} \tag{9}$$

where $\mathbf{\Phi} = \mathbf{M}\mathbf{N}^{-1}$ is a matrix of order $6 \times 6$. Detailed elements of $\mathbf{\Phi}$ are provided in Appendix A.

## 3. Solutions of multilayered poroelastic soils

In this paper, an $n$-layered soil media system with a depth $H_n$ is considered as shown in Fig. 2. The interfaces at the depths $H_i (i = 1, \ldots, n)$ are assumed to be perfectly bonded; the surface of the multilayered soil media is supposed to be free and permeable; the bottom is considered to be fixed and impermeable. Therefore, the final global stiffness matrix for the multilayered soil is established as [10].