Soil Dynamics and Earthquake Engineering 94 (2017) 83–87

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

Resistance of inner soil to the vertical vibration of pipe piles

Changjie Zheng[a,b,*], Hanlong Liu[a,b], Xuanming Ding[a,b], George Kouretzis[c]

[a] Key Laboratory of New Technology for Construction of Cities in Mountain Area (Chongqing University), Ministry of Education, Chongqing 400045, China
[b] School of Civil Engineering, Chongqing University, Chongqing 400045, China
[c] ARC Centre of Excellence for Geotechnical Science and Engineering, Faculty of Engineering and Built Environment, The University of Newcastle, Callaghan, NSW 2308, Australia

ARTICLE INFO

Keywords:
Pipe pile
Soil resistance
Inner soil
Analytical solution

ABSTRACT

The resistance offered by the inner soil to the vertical oscillation of an end-bearing pipe pile is studied analytically. The differences in the resistance to pile vibrations associated with the inner soil and the outer soil are underlined via example applications and theoretical considerations. Simplified, reduced solutions are also derived, to further investigate the wave propagation mechanisms governing the problem.

1. Introduction

The cornerstone of quantifying the dynamic response of a pile is the calculation of the dynamic resistance of the soil layers to the vibration of the pile. Various researchers have delved into this, following different approaches. For example, Parmelee et al. [1] used a non-linear discrete model to describe the dynamic stress and displacement fields of the soil. Novak [2] proposed the plane strain solution, in which soil is modeled as a series of homogeneous infinitesimally thin layers while ignoring vertical strains, and derived solutions for the frictional resistance of soil to pile vertical movements. Nevertheless, the soil vertical stress gradient in Novak's solution is neglected, which suggests that soil-pile interaction is considered only in the horizontal direction, and waves propagating in the vertical direction are ignored.

Later, Nogami and Novak [3] refined the three-dimensional continuum model to account for the soil stress gradient in the vertical direction. This model considers longitudinal waves propagating along the pile, and is certainly more rigorous than the plane strain solution [2]. However, the radial displacement of the soil associated with the vertical vibration is ignored, which suggests that vertically propagating shear waves are again omitted from the solution. To consider both vertical and radial displacements, Wang et al. [4] and Wu et al. [5] introduced two potential functions to decompose the displacements of soil. In this way, both the shear and longitudinal vertically propagating waves are considered, resulting in the true three-dimensional solution. Wang et al. [6] compared the soil frictional resistance factor obtained by their three-dimensional solution against the results of Novak [2] and Nogami and Novak [3]. They observed that there exists a secondary resonance frequency in each mode besides the dominant one found by Nogami and Novak [4]. This secondary resonance frequency is induced by the shear waves generated by the radial motion of soil.

The aforementioned studies all concern solid piles. Large diameter pipe piles, such as prestressed concrete pipe piles, large diameter steel pipe piles and large-diameter cast–in situ concrete pipe piles, are now widely used in practice [6,7]. The dynamic response of a pipe pile is different from that of a solid pile due to the existence of soil in the cavity formed by the pipe pile (i.e. inner soil). The resistance of the inner soil to the dynamic displacements of a pipe pile is also of great importance for quantifying the dynamic characteristics of the soil-pipe pile system. This note presents a study on the determination of the frictional resistance of the inner soil to the vertical vibration of an end-bearing pipe pile. Simplified, reduced solutions are also derived to investigate the error induced in the solution if we ignore the radial displacement and the vertical stress gradient of the inner soil.

2. Basic assumptions and conceptual model

The main simplifications introduced follow Nogami and Novak [3], and are as follows: (1) The inner soil consists of a linear viscoelastic layer, overlying a rigid base. (2) No normal stresses act on the free surface of the inner soil layer, and no vertical displacements occur at the bottom of the soil layer. (3) The cavity inside the pipe pile is assumed to be fully filled with soil. (4) The end-bearing pile is elastic, and remains in perfect contact with soil. Only the vertical displacement of the pile is considered, and the radial displacement at the pile–soil interface is assumed negligible. (5) The deformation of the pipe pile–soil system is small.

The conceptual model is shown in Fig. 1. A uniform vertical

* Corresponding author.
E-mail address: zcj@cqu.edu.cn (C. Zheng).

http://dx.doi.org/10.1016/j.soildyn.2017.01.002
Received 6 October 2016; Received in revised form 27 December 2016; Accepted 4 January 2017
0267-7261/

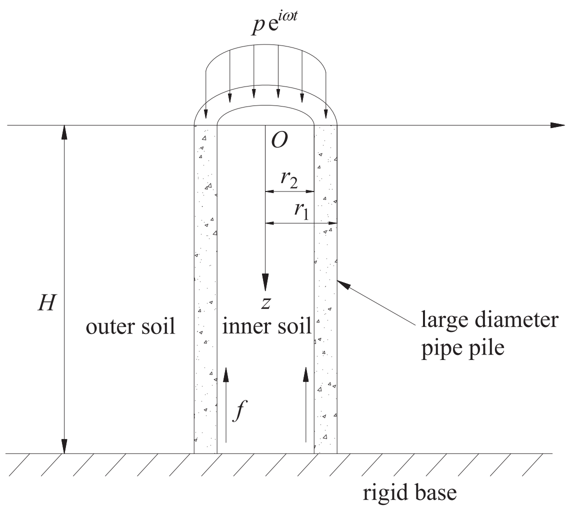

Fig. 1. Conceptual model of the end-bearing pipe pile–soil simplified system.

pressure $pe^{i\omega t}$ is applied on the pile head. H is the pipe pile length and r_1, r_2 are the outer and inner radii of the pipe pile section, respectively. Moreover, with f we denote the frictional force developing at the inner soil-pipe pile interface.

3. Governing equations and their closed-form solution

To start with, note that the solution for the outer soil of a pipe pile is essentially the same as that for the soil surrounding a solid pile, and is given in Nogami and Novak [3] and Wang et al. [4]. The following solution concerns only the reaction of the inner soil. The governing equations of the inner soil by considering both vertical and radial harmonic displacements can be written as:

$$(G+iG')\left(\nabla^2-\frac{1}{r^2}\right)u_r+[(\lambda+G)+i(\lambda'+G')]\frac{\partial e}{\partial r}=\rho\ddot{u}_r \tag{1}$$

$$(G+iG')\nabla^2u_z+[(\lambda+G)+i(\lambda'+G')]\frac{\partial e}{\partial z}=\rho\ddot{u}_z \tag{2}$$

where $\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}$; $e=\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}$ is the volumetric strain of the inner soil; u_r and u_z are the radial and vertical displacements of the inner soil, respectively; λ and G are the Lame's constants; λ' and G' are the damping; ρ is the density.

The vibration of the pile–soil system is assumed to be harmonic with a factor $e^{i\omega t}$, where ω is the cyclic frequency. The term $e^{i\omega t}$ is suppressed from all the following expressions for brevity.

Performing the differentiation $\frac{\partial(1)}{\partial r}+\frac{(1)}{r}+\frac{\partial(2)}{\partial z}$ on Eqs. (1) and (2) yields:

$$[(\lambda+2G)+i(\lambda'+2G')]\nabla^2e+\rho\omega^2e=0 \tag{3}$$

The general solution of Eq. (3) can be determined using the variable separation method:

$$e=[A_1\sin(g_1z)+B_1\cos(g_1z)][C_1K_0(q_1r)+D_1I_0(q_1r)] \tag{4}$$

where $q_1^2+\beta_1^2=g_1^2$; $\beta_1^2=\frac{\rho\omega^2}{(\lambda+2G)+i(\lambda'+2G')}$;$I_0()$ and $K_0()$ are modified Bessel functions, respectively; A_1, B_1, C_1 and D_1 are undetermined coefficients.

The boundary conditions of the inner soil are expressed as:

$$\frac{\partial u_z}{\partial z}\Big|_{z=0}=0 \tag{5}$$

$$u_z|_{z=H}=0 \tag{6}$$

$$u_r|_{r=0}<\infty \tag{7}$$

$$u_r|_{r=r_2}=0 \tag{8}$$

Substituting Eq. (4) into Eqs. (5) and (7), we obtain:

$$B_1=0 \tag{9}$$

$$C_1=0 \tag{10}$$

Then the volumetric strain e can be expressed as:

$$e=A_1I_0(q_1r)\sin(g_1z) \tag{11}$$

Substituting Eq. (11) into Eqs. (1) and (2), we obtain:

$$(G+iG')\left(\nabla^2-\frac{1}{r^2}\right)u_r+\rho\omega^2u_r=[(\lambda+G)+i(\lambda'+G')]A_1q_1K_1(q_1r)\sin(g_1z) \tag{12}$$

$$(G+iG')\nabla^2U_z+\rho\omega^2U_z=-[(\lambda+G)+i(\lambda'+G')]A_1g_1K_0(q_1r)\cos(g_1z) \tag{13}$$

The general solution of Eq. (12) can be determined by using the variable separation method:

$$u_r=[A_2\sin(g_2z)+B_2\cos(g_2z)][C_2K_1(q_2r)+D_2I_1(q_2r)] \tag{14}$$

where $q_2^2+\beta_2^2=g_2^2$; $\beta_2^2=\frac{\rho\omega^2}{G+iG'}$; A_2, B_2, C_2 and D_2 are undetermined coefficients.

The particular solution of Eq. (12) is assumed as:

$$u_r^*=\chi_1A_1K_1(q_1r)\sin(g_1z) \tag{15}$$

Substituting Eq. (15) into Eq. (12) yields:

$$\chi_1=\frac{q_1[(\lambda+G)+i(\lambda'+G')]}{\rho\omega^2-(G+iG')\beta_1^2} \tag{16}$$

The displacements and stresses of the inner soil at $r=0$ are bounded, thus

$$C_2=0 \tag{17}$$

The solution of Eq. (12) can then be written as:

$$u_r=[A_2\sin(g_2z)+B_2\cos(g_2z)]I_1(q_2r)+\chi_1A_1\sin(g_1z)I_1(q_1r) \tag{18}$$

Similarly, the solution of Eq. (13) can be obtained as:

$$u_z=[A_3\sin(g_3z)+B_3\cos(g_3z)]I_0(q_3r)+\chi_2A_1\cos(g_1z)I_0(q_1r) \tag{19}$$

where $q_3^2+\beta_2^2=g_3^2$; $\chi_2=\frac{g_1[(\lambda+G)+i(\lambda'+G')]}{(G+iG')\beta_1^2-\rho\omega^2}$; A_3 and B_3 are undetermined coefficients.

Taking into account the definition of the volumetric strain $e=\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}$ we obtain:

$$g_2=g_3,\quad q_2=q_3 \tag{20}$$

$$A_2q_2=B_3g_3,\quad B_2q_2=-A_3g_3 \tag{21}$$

Substituting Eq. (19) into Eqs. (5) and (6) yields:

$$A_3=0 \tag{22}$$

$$g_{1n}=g_{3n}=g_n=\frac{(2n-1)\pi}{2H},\ n=1,2,3,\cdots \tag{23}$$

Then the radial and vertical displacements of the inner soil can be expressed as:

$$u_r=\sum_{n=1}^{\infty}\left[\chi_{1n}A_{1n}I_1(q_{1n}r)+B_{3n}\frac{g_n}{q_{2n}}I_1(q_{2n}r)\right]\sin(g_nz) \tag{24}$$

$$u_z=\sum_{n=1}^{\infty}[\chi_{2n}A_{1n}I_0(q_{1n}r)+B_{3n}I_0(q_{2n}r)]\cos(g_nz) \tag{25}$$

Substituting Eq. (24) into Eq. (8) yields:

$$B_{3n}=\xi_nA_{1n} \tag{26}$$

where $\xi_n=-\frac{\chi_{1n}q_{2n}I_1(q_{1n}r_2)}{g_nI_1(q_{2n}r_2)}$.

The vertical displacement of the inner soil at the pile–soil interface can be written as: