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Technical Note

Transient dynamic response of a shallow buried lined tunnel in saturated soil

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ABSTRACT

Previous research has produced valuable results on the transient dynamic response of tunnels buried in full-space. However, a half-space model is of more practical interest because tunnels normally have finite buried depths. In this paper, the dynamic response of a lined tunnel is studied where the surrounding soil is described using Biot's theory and the lining is described by the theory of elastodynamics. The half-space straight boundary is approximately represented by a convex arc of large radius. In accordance with Graff's addition theorem, the general solutions in a rectangular coordinate system are converted to ones in a polar coordinate system. The solutions for displacements and stresses of both the soil and the lining as well as the pore pressure of the soil in the Laplace transform domain are derived based on boundary conditions. Time domain solutions are then obtained by the use of inverse Laplace transform. Numerical results are presented showing the distributions of peak values of ground displacements, stresses and pore pressures of the soil.

1. Introduction

Underground lined structures are sometimes subjected to transient dynamic loadings such as hydraulic fracture initiation, blasting loading and sudden excavations, which can be simplified as a suddenly applied constant load, a gradually applied step load or a triangular pulse load. These transient loadings may cause failure of the underground structures and their surrounding soil. Therefore it is very important to investigate the stress and displacement of the lined structure and the soil under the transient loadings.

Previous research has produced valuable results on the transient dynamic response of underground structures. Senjuntichai and Rajapakse [1] obtained transient solutions of a long cylindrical cavity induced by a suddenly applied constant load, a gradually applied step load and a triangular pulse load. The cavity was assumed to be buried in an infinite poroelastic medium and not to be lined. Kattis et al. [2] obtained numerical solutions for dynamic response of both the unlined and lined tunnels in an infinite poroelastic saturated soil under a harmonic wave diffraction by the boundary element method. Xie et al. [3] studied dynamic response of a partially sealed circular tunnel in viscoelastic saturated soil. Osinov [4] investigated the dynamic response of saturated granular soil induced by a blast loading on a tunnel lining. Gao et al. [5] obtained an analytical solution for transient response of a cylindrical lined cavity in a poroelastic medium. Wang et al. [6] investigated the influence of the degree of saturation on dynamic response of a cylindrical lined cavity in a nearly saturated poroelastic medium. Gao et al. [7] presented an exact solution for three-dimensional dynamic response of a cylindrical lined tunnel in saturated soil to an internal blast loading. All of these studies assumed the tunnel to be buried in a full-space, whereas a half-space model apparently is of more practical because tunnels always have finite buried depths.

In this paper, the governing equations of the soil surrounding a lined tunnel are given based on Biot's theory in a rectangular coordinate system, and the governing equations of the lining are presented based on the conventional theory of elastodynamics. The general dynamic solutions of both the surrounding soil and the lining are then obtained by the use of Laplace transform. A large radius arch is used to approximately represent the free surface boundary of the half-space. Then the general solutions in the rectangular coordinate system are transformed into solutions in the polar coordinate system by applying Graff's addition theorem [8]. By matching boundary conditions, the special solutions are derived. The peak value of ground displacement, hoop stress in the lining and pore pressure distribution

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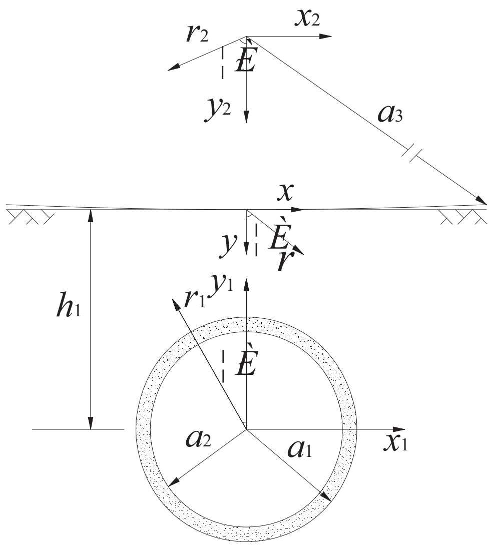

Fig. 1. Analytical model and straight boundary in the half space model.

between the lining and the surrounding soil in time domain are obtained by numerical inverse Laplace transform.

2. Governing equations

As shown in Fig. 1, a circular tunnel with infinite length is buried at the depth of h_1 in a saturated half-space. The outer and inner radii of the tunnel are a_1 and a_2, respectively. The inner surface of the lining is subjected to three types of transient loading (see Ref. [5]).

Treating the soil as a fluid-saturated poroelastic medium, the equilibrium equations of soil skeleton and the fluid are as follows:

$$(\lambda_s + \alpha^2 M + G_s)u_{j,ji} + G_s u_{i,jj} + \alpha M w_{j,ji} = \rho \ddot{u}_i + \rho_f \ddot{w}_i \quad i, j = x, y \tag{1}$$

$$\alpha M u_{j,ji} + M w_{j,ji} = \rho_f \ddot{u}_i + \rho_m \ddot{w}_i + b\dot{w}_i \quad i, j = x, y \tag{2}$$

where λ_s and G_s are Lamé constants of the saturated soil; α and M are the Biot's coefficients; u_i and w_i are the displacement of the soil skeleton and the displacement of the fluid relative to the solid; ρ is the mass density of saturated soil, ρ=(1-n_f)ρ_s+$n_f\,\rho_f$, n_f is porosity of soils, ρ_f and ρ_s are respectively the mass density of pore fluid and solid skeleton; b is the fluid viscous coupling coefficient; ρ_m is the fluid additional mass density, ρ_m=($n_f\,\rho_f$+ρ_a)/n_f^2, ρ_a is the mass density induced by fluid coupling.

By introducing potential functions :

$$u_i = \varphi_{1,\,i} + e_{ijk}\psi_{1k,j}, \quad w_i = \varphi_{2,\,i} + e_{ijk}\psi_{2k,j} \tag{3}$$

where $\varphi_1(r_1, \theta_1)$, $\psi_1(r_1, \theta_1)$ and $\varphi_2(r_1, \theta_1)$, $\psi_2(r_1, \theta_1)$ are the potential functions of soil skeleton and the fluid, respectively; e_{ijk} is the permutation tensor in rectangular coordinates.

Substituting Eq. (3) into Eqs. (1) and (2), the equilibrium equations as follows:

$$(\lambda_s + \alpha^2 M + 2G_s)\varphi_{1,ijj} + e_{ijk}\psi_{1k,ijj} + \alpha M\varphi_{2,ijj} = \rho(\ddot{\varphi}_{1,i} + e_{ijk}\ddot{\psi}_{1k,j}) + \rho_f(\varphi_{2,i} + e_{ijk}\ddot{\psi}_{2k,j}) \tag{4}$$

$$\alpha M\varphi_{1,ijj} + M\varphi_{2,ijj} = \rho_f(\ddot{\varphi}_{1,i} + e_{ijk}\ddot{\psi}_{1k,j}) + m(\ddot{\varphi}_{2,i} + e_{ijk}\ddot{\psi}_{2k,j}) + b(\dot{\varphi}_{2,i} + e_{ijk}\dot{\psi}_{2k,j}) \tag{5}$$

Applying Laplace transform to both sides of Eqs. (4) and (5), denoting $\overline{\varphi}_1 = L[\varphi_1]$, $\overline{\varphi}_2 = L[\varphi_2]$, $\overline{\psi}_1 = L[\psi_1]$, $\overline{\psi}_2 = L[\psi_2]$ as the Laplace transform of $\varphi_1(r_1, \theta_1)$, $\varphi_2(r_1, \theta_1)$, $\psi_1(r_1, \theta_1)$, $\psi_2(r_1, \theta_1)$, the governing equations of the soil can be written as (the non-dimensionalized quantities with respect to length and time by selecting the inner radius of the tunnel a_2 as a unit of length and $a_2(\rho/G_s)^{0.5}$ as a unit of time, see Ref. [5]):

$$(\lambda_c + 2)\nabla^2\overline{\varphi}_1 + \alpha M^*\overline{\varphi}_2 = s^2\overline{\varphi}_1 + \rho^* s^2\overline{\varphi}_2,$$
$$\alpha M^*\nabla^2\overline{\varphi}_1 + M^*\overline{\varphi}_2 = \rho^* s^2\overline{\varphi}_1 + (\rho_m{}^* s^2 + bs)\overline{\varphi}_2 \tag{6}$$

$$\nabla^2\overline{\psi}_1 = s^2\overline{\psi}_1 + \rho^* s^2\overline{\psi}_2, \quad \rho^* s^2\overline{\psi}_1 + (\rho_m{}^* s^2 + b^* s)\overline{\psi}_2 = 0 \tag{7}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$; $\lambda_c = \lambda_s{}^* + \alpha M^*$; $\lambda_s{}^* = \frac{\lambda_s}{G_s}$; $M^* = \frac{M}{G_s}$; $\rho^* = \frac{\rho_f}{\rho}$; $\rho_m{}^* = \frac{\rho_m}{\rho}$; $b^* = \frac{a_2 b}{\sqrt{\rho G_s}}$; t^*=t/[$a_2(\rho/G_s)^{0.5}$], t is time.

Following the procedures in Gao et al. [5], the general solutions of the governing Eqs. (6)–(7) can be derived as:

$$\overline{\varphi}_1 = B_3(s)I_0(\beta_3 r^*) + B_4(s)K_0(\beta_4 r^*), \quad \overline{\varphi}_2 = C_3(s)I_0(\beta_3 r^*) + C_4(s)K_0(\beta_4 r^*) \tag{8}$$

$$\overline{\psi}_1 = D_5(s)K_0(\beta_5 r^*), \quad \vec{\psi}_2 = m_7\overline{\psi}_1, \quad m_7 = -(\rho^* s^2)/(\rho_m{}^* s^2 + b^* s) \tag{9}$$

where $B_3(s)$, $B_4(s)$, $C_3(s)$, $C_4(s)$ and $D_5(s)$ are undetermined coefficients, I_0 and K_0 are the modified Bessel functions of the first and second kinds of order 0, respectively; s is the Laplace transform parameter; β_3, β_4, β_5 are the dimensionless wave numbers associated with the two dilatational waves and shear wave, respectively. They can be written as follows:

$$\beta_3{}^2 = \frac{m_3 + \sqrt{m_3{}^2 - 4m_4}}{2}, \quad \beta_4{}^2 = \frac{m_3 - \sqrt{m_3{}^2 - 4m_4}}{2},$$
$$\beta_5{}^2 = \frac{(\rho_m{}^* s^2 + b^* s)s^2 - \rho^{*2} s^4}{(\rho_m{}^* s^2 + b^* s)} \tag{10}$$

$$m_3 = \frac{(\lambda_c + 2)(\rho_m{}^* s^2 + b^* s) - s^2 M^* + 2\alpha M^*\rho^* s^2}{\lambda_s{}^* M^* + 2M^*}, \; m_4 = \frac{(\rho_m{}^* s^2 + b^* s)s^2 - \rho^{*2} s^4}{\lambda_s{}^* M^* + 2M^*}, \quad m_7 = -\frac{\rho^* s^2}{(\rho_m{}^* s^2 + b^* s)} \tag{11}$$

The lining of the cavity is treated as an elastic medium. The governing equations can be obtained as follows by linear elastic wave theory,

$$G u^L_{i,jj} + (\lambda + G)u^L_{j,ji} = \rho_L \ddot{u}_i^L \tag{12}$$

where λ and G are respectively the lining elastic constants; ρ_L is the mass density of the lining; $u_i{}^L$ and $\ddot{u}_i{}^L$ are respectively the lining displacement and acceleration.

Similarly, the general solutions of the lining can be written as:

$$\overline{\varphi} = A_6(s)I_0(\beta_6 r^*) + B_6(s)K_0(\beta_6 r^*) \tag{13}$$

$$\overline{\psi} = A_7(s)I_0(\beta_7 r^*) + B_7(s)K_0(\beta_7 r^*) \tag{14}$$

where $A_6(s)$, $A_7(s)$, $B_6(s)$ and $B_7(s)$ are the undetermined coefficients; β_6, β_7 are the dimensionless wave numbers associated with the compression wave and shear wave and can respectively be expressed as $\beta_6{}^2$=$s^2(c^*)^2$ and $\beta_7{}^2$=$s^2(c_s{}^*)^2$, in which $(c^*)^2$=$(\lambda^*$+$G^*)/\rho_L{}^*$, $(c_s{}^*)^2$=$G^*/\rho_L{}^*$, λ^*=λ/G_s, G^*=G/G_s, $\rho_L{}^*$=ρ_L/ρ. Here c^* and $c_s{}^*$ are the non-dimensionalized velocity of compression wave and shear wave, respectively.

3. Coordinate transformation

For the half-space straight boundary, the boundary is represented as a convex arc with a large radius a_3 (a_3=6000 a_2), as shown in Fig. 1.

Using addition theorem by Graff [8], a set of general solutions in the rectangular coordinate system are transformed into solutions in the polar coordinate system:

$$Z_n(\beta r_2)\cos(n\theta_2) = \sum_{n=-\infty}^{\infty} Z_{n+m}(\beta D)J_m(\beta r_1)\cos(n\theta_1), \quad (r_2 < D) \tag{15}$$

where $Z_n(*)$ is the functions of $I_n(*)$ or $K_n(*)$; D is the distance between two origins of the coordinate systems.