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Efficient solution for the diffraction of elastic SH waves by a wedge: Performance of various exact, asymptotic and simplified solutions

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A R T I C L E I N F O

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A B S T R A C T

The diffraction of horizontally polarized shear waves by a semi-infinite wedge in frequency and time domains is studied. In particular, this work focus on the performance of different solutions, including the classical contributions from Macdonald, Sommerfeld and Kouyoumjian & Pathak. In addition, two fully analytical, simplified solutions are proposed using arguments from the so-called geometrical theory of diffraction. The main advantage of the two proposed solutions is the fact that the resulting solutions can be scaled to problems with arbitrary and complex geometries. Moreover, it is found that one of the proposed new solutions is highly efficient in terms of accuracy and computational speed as compared to alternative formulations (approximately 1000 times faster than the Macdonald and Kouyoumjian & Pathak solutions), thus, this important characteristic renders this solution ideal for implementation in GPUs (Graphics Processor Units) for multiscale modeling applications.

1. Introduction

The scattering of elastic waves by arbitrarily shaped surfaces that generate geometric effects is important in many practical applications, such as soil exploration when searching for natural sources, geotechnical applications, rock mechanics and earthquake engineering. For a comprehensive literature review on the diffraction problem for wedges and its applications the reader may consult the work by Jaramillo et al. [1].

In this work, the performance of different solutions for a wedge-shaped surface with incident, horizontally polarized, shear (SH) waves is studied. The problem of elastic wave propagation in a wedge is relevant from the viewpoint of the geometrical theory of diffraction [2,3] because it allows for scaling to more complex problems from the application of basic problems called "canonical problems" [1]; these ideas have been widely used in other areas due to its scalability and relative ease in numerical implementations and its sufficient accuracy for engineering applications [4,5].

This paper is organized as follows. In Section 2, the geometric field is briefly presented. In Section 3, the Macdonald exact solution in series expansion in terms of Bessel functions for the total field of a wedge is given. In Section 4, the Sommerfeld asymptotic solution, or the Keller solution, is described, which is a non-uniform solution at high frequency ($k_s r \gg 1$) because it is not valid near the reflection and transmission boundaries. In Sections 5, 6 and 7, the solution of Kouyoumjian & Pathak and simplified solutions 1 and 2 are presented, which are uniform solutions. The Sommerfeld (asymptotic), Kouyoumjian & Pathak, and simplified solutions 1 and 2 are based on the ideas of the geometrical theory of diffraction. Finally, a comparative study of accuracy and computational performance of these solutions is performed.

2. Geometric solution

First, a geometric description of the two-dimensional problem of a plane harmonic SH wave falling on a clamped or traction-free wedge is provided in this section. Fig. 1 shows the geometric solution, where the discontinuity of the displacement field at the incident and reflection boundaries can be observed.

Fig. 1 shows the three wedge-shaped regions defined by the half-plane and the directions of the reflected and transmitted rays:

$$\text{Reflection region:} 0 \le \phi \le \pi - \phi_0$$

$$\text{Transmission region:} \pi - \phi_0 \le \phi \le \pi + \phi_0 \text{Shadow region:} \pi + \phi_0 \le \phi \le \nu\pi. \quad (1)$$

Now, an incident plane SH wave falling at ϕ_0 is considered. The time factor $e^{i\omega t}$ is assumed and is omitted, and the g superscript

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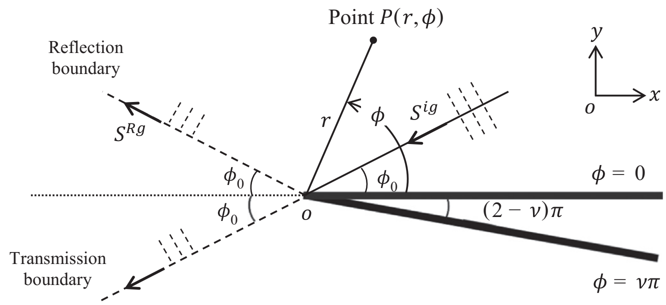

Fig. 1. Geometric displacement field of a plane SH wave falling on a wedge, where r and ϕ are cylindrical coordinates.

indicates the geometric field:

$$S^{ig} = u_0 e^{-i\vec{k}_s^{ig}\cdot\vec{r}}$$

$$=u_0 e^{ik_s(x\cos\phi_0 + y\sin\phi_0)} = u_0 e^{ik_s r\cos(\phi-\phi_0)}, \tag{2}$$

which, after incidence in a plane-discontinuity, generates a reflected plane SH wave:

$$S^{Rg} = u_0 R_{SH} e^{-i\vec{k}_s^{Rg}\cdot\vec{r}}$$

$$=u_0 R_{SH} e^{ik_s(x\cos\phi_0 - y\sin\phi_0)} = u_0 R_{SH} e^{ik_s r\cos(\phi+\phi_0)}, \tag{3}$$

where $k_s = 2\pi/\lambda_s = \omega/c_s = 2\pi f/c_s$ is the wavenumber for secondary waves, ω is the angular frequency, f is the frequency, and c_s and λ_s are the propagation velocity and the wavelength for secondary waves, respectively. The reflection coefficient R_{SH} is

$$R_{SH} = \begin{cases} -1, \text{for a clamped boundary (Dirichlet condition)} \\ 1, \text{for a traction-free boundary (Neumann condition)} \end{cases} \tag{4}$$

Finally, the total geometric solution can be defined as

$$\text{Reflection region:} u^g = S^{ig} + S^{Rg}$$

$$\text{Transmission region:} u^g = S^{Rg} \text{Shadow region:} u^g = 0. \tag{5}$$

3. Macdonald exact solution

Second, the exact solution of Macdonald to the problem of a plane SH wave diffracted by a wedge-shaped medium is given by [6,7]

$$u_{exact}(r,\phi) = S^i + S^R \tag{6}$$

with

$$S^i = \frac{2u_0}{\nu}\sum_{n=0}^{\infty}\epsilon_n e^{i\pi\eta_n/2} J_{\eta_n}(k_s r)\cos[\eta_n(\phi-\phi_0)] S^R = \frac{2u_0 R_{SH}}{\nu}\sum_{n=0}^{\infty}\epsilon_n e^{i\pi\eta_n/2} J_{\eta_n}(k_s r)\cos[\eta_n(\phi+\phi_0)] \tag{7}$$

where $\eta_n = n/\nu$, R_{SH} is the reflection coefficient defined in (4) and

$$\epsilon_n = \begin{cases} \epsilon_0 = \epsilon_1 = \epsilon_2 = \ldots = 1, & \text{for a clamped boundary} \\ \epsilon_0 = 1/2, \quad \epsilon_1 = \epsilon_2 = \ldots = 1, & \text{for a traction-free boundary} \end{cases} \tag{8}$$

Here, it is important to note that the Macdonald series expansion has a very slow convergence for large values of $k_s r$, that is, when $k_s r \gg 1$.

4. Sommerfeld asymptotic solution or Keller solution (non-uniform)

Third, the Sommerfeld asymptotic solution for high frequency [6] (also called the non-uniform solution), which was also achieved by Keller through the geometrical theory of diffraction [2], is given by

$$u_{non}(r,\phi) = S^i_{non} + S^R_{non} \tag{9}$$

where

$$S^i_{non} = u_0 e^{(ik_s r\cos(\phi-\phi_0))} h[\cos(\tfrac{\phi-\phi_0}{2})] + u_0 \frac{e^{(-ik_s r)}}{\sqrt{k_s r}} D^i_{non},$$
$$S^R_{non} = u_0 R_{SH} e^{ik_s r\cos(\phi-\phi_0)} h[\cos(\tfrac{\phi+\phi_0}{2})] + u_0 R_{SH}\frac{e^{-ik_s r}}{\sqrt{k_s r}} D^R_{non}, \tag{10}$$

where $h(X)$, the Heaviside unit-step function, is defined as

$$h(X) = \begin{cases} 1 & \text{if } X \geq 0 \\ 0 & \text{if } X < 0 \end{cases}; \tag{11}$$

and

$$D^i_{non} = \frac{(e^{(-i\pi/4)}\sin(\frac{\pi}{\nu}))}{\nu\sqrt{2\pi}}\left[\frac{1}{\cos\left(\frac{\pi}{\nu}\right) - \cos\left(\frac{\phi-\phi_0}{2}\right)}\right],$$
$$D^R_{non} = \frac{e^{-i\pi/4}\sin\left(\frac{\pi}{\nu}\right)}{\nu\sqrt{2\pi}}\left[\frac{1}{\cos\left(\frac{\pi}{\nu}\right) - \cos\left(\frac{\phi+\phi_0}{2}\right)}\right] \tag{12}$$

are the first-order diffraction coefficients of the Sommerfeld asymptotic solution. This asymptotic solution can also be written and understood as

$$u_{non}(r,\phi) = u^g + u^d_{non} \tag{13}$$

where

$$u^d_{non} = u_0\frac{e^{-ik_s r}}{\sqrt{k_s r}} D^i_{non} + u_0 R_{SH}\frac{e^{-ik_s r}}{\sqrt{k_s r}} D^R_{non} \tag{14}$$

is the diffracted field of the non-uniform solution and

$$u^g = u_0 e^{ik_s r\cos(\phi-\phi_0)} h\left[\cos\left(\frac{\phi-\phi_0}{2}\right)\right] + u_0 R_{SH} e^{ik_s r\cos(\phi+\phi_0)} h\left[\cos\left(\frac{\phi+\phi_0}{2}\right)\right]$$
$$= S^{ig} h\left[\cos\left(\frac{\phi-\phi_0}{2}\right)\right] + S^{Rg} h\left[\cos\left(\frac{\phi+\phi_0}{2}\right)\right] \tag{15}$$

is the same geometric field defined in (5) in compact form, representing the total field. The signs of $\cos(\frac{\phi\pm\phi_0}{2})$ within the regions defined in (1) and (5) are as follows:

$$\text{Reflection región:} u_{som} = S^{ig} + S^{Rg} + u^d_{non}; \cos\left(\frac{\phi-\phi_0}{2}\right) > 0, \quad \cos\left(\frac{\phi+\phi_0}{2}\right) > 0$$

$$\text{Transmission region:} u_{som} = S^{ig} + u^d_{non}; \cos\left(\frac{\phi-\phi_0}{2}\right) > 0,$$
$$\cos\left(\frac{\phi+\phi_0}{2}\right) < 0 \text{Shadow region:} u_{som} = u^d_{non}; \cos\left(\frac{\phi-\phi_0}{2}\right) < 0, \cos\left(\frac{\phi+\phi_0}{2}\right) < 0. \tag{16}$$

In Fig. 2, the wavefront pattern of the Sommerfeld asymptotic solution is shown for $0 < \phi_0 < \pi/2$.

5. Kouyoumjian & Pathak solution (uniform)

Fourth, the Kouyoumjian & Pathak solution for high frequency [1,3] (also called the uniform solution) is an improvement of the Sommerfeld asymptotic solution based on the Keller method [2], achieved by proposing diffraction coefficients that remain valid near the reflection and transmission boundaries, where the coefficients in (12) fail. Then, this solution is given by

$$u_{uni}(r,\phi) = u^g + u^d_{uni} \tag{17}$$

where u^g is the same geometric field defined in (15) and