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Soil and topographic effects on ground motion of a surficially inhomogeneous semi-cylindrical canyon under oblique incident SH waves

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ABSTRACT

To elucidate the ground motion amplification due to combined soil and topographic effects, an analytical formulation in the framework of classical elastodynamics is derived for the scattering of oblique incident plane SH waves by a semi-cylindrical canyon covered by a local inhomogeneous soil layer with a radially-varying modulus. Allowing the shear modulus of the finite soil layer covering the circular surface of the canyon as a power of the radial distance, the governing equation of motion for the anti-plane shear problem is derived and solved analytically by the method of wave function expansion. The ground motions for both of the homogeneous and inhomogeneous canyons under oblique incident waves can be computed efficiently and a comprehensive set of numerical examples are presented as illustrations. The degree of inhomogeneity of the soil layer and its thickness are found to affect the magnitude and the pattern of ground motion amplification of the cylindrical canyon surface depending on the frequency content, the irregular topography and obliquity of the wave incidence.

1. Introduction

It is well known that the earthquake ground motion can be significantly modified due to local soil stratigraphy and topography as both can induce significant amplifications of the seismic ground motions [1,2]. As the magnitude and spatial variation of ground motions are key input to the seismic design of critical structures, the subject of site effects has been a focus area in earthquake engineering and soil dynamics. In engineering seismology, the subject is approached from the viewpoint of determining the ground motion pattern over the length scale of a geological setting under incident seismic waves. Domain-type numerical methods such as finite differences and finite elements are flexible in modeling complex geometries and heterogeneous materials of realistic geological settings while boundary element formulations are adroit for a rigorous treatment of unbounded-domain wave propagation phenomena. Aimed to combine the appealing features of the two techniques, hybrid finite element-boundary element approaches have also been developed for some large-scale site modeling problems [3]. In geotechnical engineering, a common focus is on how the soil's material condition can alter the seismic ground motion at the upper soil region where structures are founded [4]. For the case of level ground, 1D ground response analysis of horizontal soil layers subjected to the vertically propagating shear waves has been commonly used in geotechnical earthquake engineering [5]. Such a simplification is, however, often found to be insufficient in explaining measured free-field or structure's responses or the observed damage pattern when the terrain and geological setting of the region is more complicated. Investigation of such events [6–9] has demonstrated that there is significant presence of topological, seismological, as well as geotechnical factors.

On the influence of a site's topography on the ground motion, 2D models of valley effects and 3D basin effects have been attracting much interest in geotechnical earthquake engineering in recent years [10–15]. Most of those studies focused on the total seismic response of 2D alluvial valleys or 3D sediment-filled basins of uniform properties. It has also been commonly taken that concave topographies such as canyons are capable of reducing ground motions [16,17], while a softer overlying soil region on them can amplify ground motions because of the impedance contrast. The separation of the contribution of local topography from other causes, such as stratigraphy and near surface weathering is not an easy undertaking [18]. Prompted by observational evidence during the Athens 1999 earthquake, attempts to dissect the relative contribution of soil and topographic effects on the uneven damage distribution have been largely based on numerical methods [19–23]. For example, Gatmiri and his co-workers [24,25] developed a hybrid numerical method, combining finite elements and boundary

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elements in an attempt to differentiate the soil and topographic effects on seismic site amplification. Tripe et al. [26] studied the relationship and interaction between soil and topographic amplification for the case of a slope based on the finite element analysis.

Aiming to advance the understanding of soil and topographic effects, a rigorous analytical solution for a surficial inhomogeneous semi-cylindrical canyon subjected to incident plane SH waves is presented in this paper. The problem configuration is a generalization of the canonical 2D homogeneous semi-cylindrical canyon model [27] by allowing a surficial soil layer to be present on its curved surface [28–33] to represent surficial weathering or soil sedimentation to a finite depth from water flows or other geological mechanisms [34]. To be a more realistic representation of typical soil types and behavior in such conditions, the shear modulus of the surficial soil layer is taken to be a power function of the radial distance from the axis of cylindrical geometry as in [35,36]. In what follows, it is shown that the formulation is amenable to a rigorous analytical treatment by means of a wave function expansion method and a comprehensive parametric study is presented to illustrate the key features of the 2D wave scattering-diffraction problem.

2. Canyon model and excitation

The seismic canyon model in this study is depicted in Fig. 1. It shows the cross-section of a uniform semi-cylindrical canyon of half-width a with an inhomogeneous surficial soil layer whose thickness is b-a and shares the same center of the canyon. The contact interface between the surficial layer (Region 1) and the underlying grooved half-space of soil or rock (Region 2) is assumed to be perfectly bonded. The material property of the medium in the Region 2 is assumed to be homogeneous and linearly elastic. The mass densities of the media in both Regions 1 (ρ_1) and 2 (ρ_2), the shear modulus μ_2 and shear wave velocity c_2 of the media in the Region 2 are all taken to be constant and isotropic. In contrast, the surficial inhomogeneous layer (Region 1) is assumed to be elastic, has constant mass density ρ_1 but an inhomogeneous shear modulus μ_1. For generality, the shear wave velocity c_1 is assumed to have a power-law variation with respect to the radial depth r measured from the center O in Fig. 1. Analytically, the shear wave velocity model for the canyon-inhomogeneous layer system can be expressed as

$$c_1(r) = c_0\left(\frac{r}{a}\right)^{\beta}, \quad a \le r \le b, \quad \beta \ge 0, \tag{1a}$$

for the surficial layer and

$$c_2 = c_1(b) = c_0\left(\frac{b}{a}\right)^{\beta}, \quad r > b, \quad \beta \ge 0, \tag{1b}$$

for the underlying homogeneous half-space, with β parameterizing the degree of inhomogeneity of the surficial soil layer, and c_0 being the shear wave velocity at the circular surface at $r=a$. Eq. (1a) also indicates that the shear modulus of Region 1 is a function of r with $\mu_1(r) = \rho_1 c_1^2(r)$. For $\beta \ge 0$, the soil layer will have either a shear modulus that is increasing with the radial depth r or one that is homogeneous.

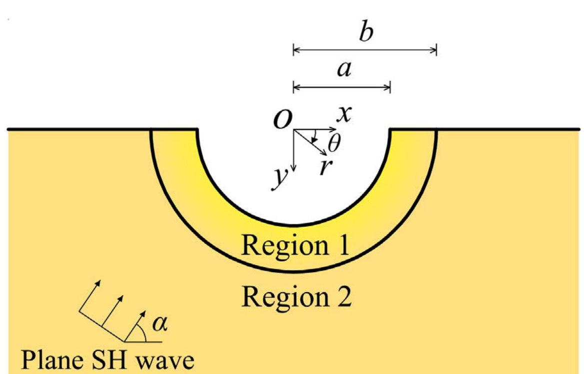

Fig. 1. Cross-section of a semi-cylindrical canyon with a radially inhomogeneous surface layer.

The excitation of the model is a steady train of time-harmonic plane SH waves with an incident angle α, circular frequency ω, and a horizontal displacement in the z-direction. Both the Cartesian and polar coordinate systems are employed as shown in Fig. 1. The origins of Cartesian coordinate system (x, y) and polar coordinate system (r, θ) are both set at the axis of revolution of the semi-cylindrical canyon. The angle θ is measured from the horizontal x-axis clockwise towards the y-axis.

3. Mathematical formulation

3.1. Equation of motion and wave field of the surficial inhomogeneous soil layer

The gover ning differential field equation for the finite surficial soil layer with zero body-force field where its Lame's constants λ_1 and μ_1 are varying functions of space can be expressed (Pak & Guzina [37]) as

$$(\lambda_1 + 2\mu_1)\nabla(\nabla\cdot\boldsymbol{u}) - \mu_1\nabla\times\nabla\times\boldsymbol{u} + (\nabla\cdot\boldsymbol{u})\nabla\lambda_1 + (\nabla u + \nabla\boldsymbol{u}^T)\nabla\mu_1 = \rho_1\ddot{\boldsymbol{u}}, \tag{2}$$

where $\boldsymbol{u}$ is the displacement vector in the cylindrical coordinate system (r, θ, z) shown in Fig. 1. The displacement vector $\boldsymbol{u}$ of the surficial inhomogeneous soil layer in anti-plane motion in the z-direction can be reduced to only its z-component w_1 (r, θ, t). With

$$\nabla\cdot\boldsymbol{u} = 0, \quad \nabla\times\nabla\times\boldsymbol{u} = \left(0, 0, -\frac{\partial^2 w_1}{\partial r^2} - \frac{1}{r}\frac{\partial w_1}{\partial r} - \frac{1}{r^2}\frac{\partial^2 w_1}{\partial\theta^2}\right),$$
$$(\nabla u + \nabla\boldsymbol{u}^T)\nabla\mu_1 = \left(0, 0, \frac{\partial\mu_1}{\partial r}\frac{\partial w_1}{\partial r}\right), \tag{3}$$

the equation of motion for the radially inhomogeneous soil layer can be expressed as

$$\mu_1(r)\left(\frac{\partial^2 w_1}{\partial r^2} + \frac{1}{r}\frac{\partial w_1}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w_1}{\partial\theta^2}\right) + \frac{\partial\mu_1(r)}{\partial r}\frac{\partial w_1}{\partial r} = \rho_1\ddot{w}_1, \quad a \le r \le b,$$
$$0 \le \theta \le \pi, \tag{4}$$

which becomes

$$\frac{\partial^2 w_1}{\partial r^2} + \frac{1+2\beta}{r}\frac{\partial w_1}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w_1}{\partial\theta^2} = \frac{1}{c_1^2}\ddot{w}_1, \quad a \le r \le b, \quad 0 \le \theta \le \pi \tag{5}$$

for the power-law variation of the shear wave velocity as Eq. (1a). In the frequency domain, one may write $w_1(r, \theta, t)$ as $w_1(r, \theta)\, e^{-i\omega t}$ with ω being the circular frequency, and Eq. (5) can be further reduced to

$$\frac{\partial^2 w_1}{\partial r^2} + \frac{1+2\beta}{r}\frac{\partial w_1}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w_1}{\partial\theta^2} + k_1^2 w_1 = 0, \quad a \le r \le b, \quad 0 \le \theta \le \pi, \tag{6}$$

where

$$k_1(r) = \frac{\omega}{c_1(r)} = \left(\frac{\omega}{c_0}\right)\left(\frac{r}{a}\right)^{-\beta} = k_0\left(\frac{r}{a}\right)^{-\beta} \tag{7}$$

is the radially-varying wave-number of the medium in the surficial inhomogeneous soil layer, and k_0 is the reference wave-number for its surface region.

Seeking an eigenfunction expansion of $w_1(r, \theta)$ with a periodicity of 2π with respect to θ by writing

$$w_1(r, \theta) = R(r)\Phi(\theta), \tag{8}$$

one finds that Eq. (6) leads to

$$\frac{r^2 R''(r)}{R(r)} + \frac{(1+2\beta)rR'(r)}{R(r)} + k_0^2 a^{2\beta} r^{2-2\beta} = -\frac{\Phi''(\theta)}{\Phi(\theta)} = n^2, \tag{9}$$