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Liquefaction potential evaluations by energy-based method and stress-based method for various ground motions: Supplement

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ABSTRACT

Energy-Based Method for liquefaction potential evaluation was previously proposed and applied to simple soil models and case history sites to show its general usability for a variety of seismic motions. The key of the proposed method is to compare upward wave energy with energy capacity for liquefaction in each layer, though the theoretical background in the energy comparison was not fully addressed in the previous paper. In this supplement, wave energy in upward propagating SH-wave is formulated together with associated dissipated energy, and how to compare it with liquefaction energy capacity is discussed in a simplified evaluation procedure of EBM incorporating cyclic loading soil test data in the laboratory. An additional case study is also conducted to know the effect of the simplification on evaluation results.

1. Introduction

In a previous paper by Kokusho and Mimori [1], an energy-based liquefaction evaluation method (EBM) previously developed by Kokusho [2] was applied to a uniform sand model shaken by seismic motions recorded at different sites during different magnitude earthquakes. It was also employed in evaluating actual liquefaction case histories where geotechnical data as well as recorded earthquake motions were available. The results demonstrated that, for several ground motions employed, EBM tends to give basically compatible results with the stress-based liquefaction evaluation method (SBM), if appropriate stress reduction coefficients r_n are chosen in accordance with earthquake magnitudes. However, for exceptional ground motions in which acceleration is too large/small compared to small/large energy, a gap widens between EBM and SBM, which is too large for the coefficient r_n in SBM to be adjustable.

The common basis of energy-based liquefaction evaluations so far proposed is that the dissipated energy in soils during seismic loading is a key parameter governing pore-pressure buildup or induced strain during liquefaction. Among them, in the EBM proposed by Davis & Berrill [3] the dissipated energy in liquefiable sand was not actually quantified to compare with input seismic wave energies in individual layers. Instead, seismic energies evaluated by empirical formulas using earthquake magnitudes and seismic source distances presumably at bedrock levels were directly plotted versus SPT blow counts in liquefaction case histories to empirically develop boundary curves separating liquefaction/non-liquefaction on the SPT N-value ~ energy plane. On the other hand in EBM proposed by Kazama et al. [4], seismic wave energy was not calculated, but dissipated energies were calculated at individual layers in one-dimensional soil response analyses using design motions to compare with threshold energies for liquefaction.

On the other hand, in EBM developed by Kokusho [2] unlike other EBMs, seismic wave energies are calculated as energy demands in individual layer units and compared with energy capacities correlated with dissipated energies. To the best of the author's knowledge, seismic wave energies have scarcely been directly quantified in engineering designs to compare with energy capacities in soils or structures. In this regard, theoretical backgrounds were not fully discussed in the previous papers [1,2] on how to evaluate seismic wave energies of design motions and how to compare with dissipated energies based on laboratory soil test results.

In this supplement, seismic wave energy and corresponding dissipated energy in upward propagating SH-wave are first formulated considering the effect of the boundary condition at the ground surface. Then theoretical backgrounds in comparing the upward wave energy with the liquefaction energy capacity using cyclic loading soil test data in the laboratory are discussed together with some approximations introduced to simplify the evaluation. Finally, an additional case study is conducted to examine the applicability of the simplified procedure in this energy-based method.

2. Energy In upward propagating SH-wave

In this energy method, the upward seismic wave energy is directly compared with the energy capacity corresponding to a particular state

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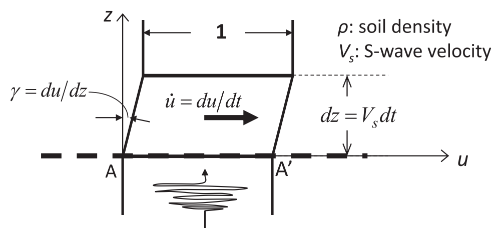

Fig. 1. Energy in upward propagating SH-wave.

of liquefaction such as 100% pore-pressure buildup or a certain induced strain amplitude in each layer unit. Among the SH-wave energies flowing up and down in the ground, the upward energy is solely considered in the comparison, because the downward energy originally constitutes a part of the upward energy if the cumulative energy is concerned as already stated in the previous paper [2].

The energy of upward propagating SH-wave with the wave velocity V_S passing through a horizontal plane A-A′ of a unit area illustrated in Fig. 1 is formulated as follows. Kinetic energy increment in a soil element of a unit horizontal area and small thickness $dz = V_s\Delta t$ (travel distance in a small time increment Δt) having particle velocity $\dot{u}$ can be expressed as

$$\Delta E_k = \frac{1}{2}\rho V_s \Delta t(\dot{u})^2 \tag{1}$$

Strain energy increment stored in the same soil element is expressed by shear stress $\tau = G\gamma$ and shear strain γ and using $\gamma = -\dot{u}/V_s$ as;

$$\Delta E_e = \int_0^\gamma V_s \Delta t \tau d\gamma = V_s \Delta t G \int_0^\gamma \gamma d\gamma = \frac{1}{2}\rho V_s^3 \Delta t \gamma^2 = \frac{1}{2}\rho V_s \Delta t(\dot{u})^2 \tag{2}$$

Hence, $\Delta E_k = \Delta E_e$, and the wave energy increment passing through the area in a time increment Δt is their sum expressed as;

$$\Delta E = \Delta E_k + \Delta E_e = \rho V_s \Delta t(\dot{u})^2 \tag{3}$$

The cumulative energy in a time interval $t=t_1\text{~}t_2$ can be expressed as the sum of the two energies of equal amounts (Timoshenko and Goodier [5], Sarma [6]).

$$E = E_k + E_e = \rho V_s \int_{t_1}^{t_2} (\dot{u})^2 dt \tag{4}$$

Thus, the wave energy of unilaterally propagating wave consists of 50% kinetic and 50% strain energy, and its dimension is energy per unit area.

Let us consider now upward harmonic SH-wave shown in Fig. 2 propagating in the z-axis (upward direction) with time t in a viscoelastic medium as;

$$u = B \sin \omega(t - (z/V_s^*)) \tag{5}$$

Here, B=a wave amplitude, ω=angular frequency, and V_s^*(complex S-wave velocity considering non-viscous damping) can be written as;

$$V_s^* = \sqrt{(G + iG')/\rho} = V_s(1 + \tan^2\delta)^{1/4} e^{i\delta/2} \tag{6}$$

where $G + iG'$=complex shear modulus with real and imaginary parts and ρ=soil density. The phase delay angle δ is defined by using G and G' and also correlated with damping ratio D as;

$$\delta = \tan^{-1}(G'/G) = \tan^{-1}(2D). \tag{7}$$

Eq. (5) is also written in the following form

$$u = Be^{-\beta z} \sin \omega(t - z/V_s') \tag{8}$$

where modified S-wave velocity is defined as

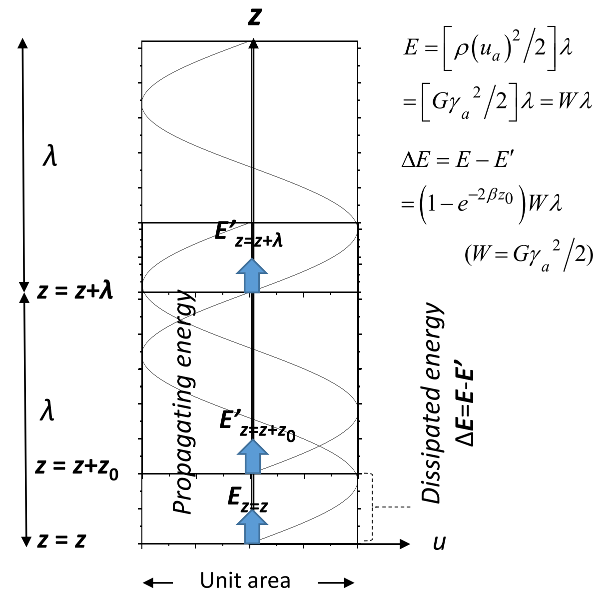

Fig. 2. Wave energy and dissipated energy in upward propagating harmonic SH-wave with wave length λ.

$$V_s' = \frac{V_s}{(\cos\delta)^{1/2}(\cos(\delta/2))} \tag{9}$$

and can be approximated as $V_s' \approx V_s^*$ for $\delta << 1.0$. The wave attenuation coefficient β is defined as

$$\beta = (\cos\delta)^{1/2}(\sin(\delta/2))(\omega/V_s) = (\tan(\delta/2))(\omega/V_s') \tag{10}$$

which can be approximated for $\delta << 1.0$ as

$$\beta \approx (\omega \tan\delta)/(2V_s) = \omega D/V_s \tag{11}$$

If Eq. (8) is substituted into Eq. (4), and integrated for one period of the harmonic wave (t=0~$2\pi/\omega$), the energy in one wave length $\lambda = 2\pi V_s/\omega$ can be obtained as;

$$E = \rho V_s \int_0^{2\pi/\omega} (\dot{u})^2 dt = \rho(\omega^2 B^2 e^{-2\beta z}/2)\lambda \tag{12}$$

Using the amplitude of particle velocity as

$$\dot{u}_a = |\omega B e^{-\beta z} \cos \omega(t - z/V_s)|_{\max} = \omega B e^{-\beta z} \tag{13}$$

and the shear strain amplitude, wherein $\delta << 1.0$ is assumed, as

$$\gamma_a \approx -Bke^{-\beta z}|\cos \omega(t - z/V_s)|_{\max} = -Bke^{-\beta z} = -\dot{u}_a/V_s \tag{14}$$

the upward wave energy in Eq. (12) can be expressed as

$$E = \rho(\omega^2 B^2 e^{-2\beta z}/2)\lambda = [\rho(\dot{u}_a)^2/2]\lambda = [G\gamma_a^2/2]\lambda \tag{15}$$

Hence the energy density per wave length in the upward wave is

$$E/\lambda = \rho(\dot{u}_a)^2/2 = G\gamma_a^2/2 = W \tag{16}$$

where W is equal to the maximum strain energy in a cyclic loading test of an elastic material of shear modulus $G = \rho V_s^2$ with the amplitude γ_a.

As already explained, this energy density W is carried evenly by $W/2$ each in kinetic and strain energies. This is confirmed by the following calculations that the kinetic energy of the upward wave with the amplitude $\dot{u}_a$ averaged over one period T is written as;

$$\frac{1}{T}\int_0^T (\rho\dot{u}^2/2)dt = \frac{1}{T}\int_0^{2\pi/\omega} \frac{\rho\dot{u}_a^2}{2}\cos^2\omega(t - z_0/V_s)dt = W/2 \tag{17}$$

and the strain energy of the same wave with the amplitude γ_a averaged over one period T is written as;