Soil Dynamics and Earthquake Engineering 89 (2016) 128–135

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

A new series solution method for two-dimensional elastic wave scattering along a canyon in half-space

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ARTICLE INFO

Article history:
Received 21 January 2016
Received in revised form
2 July 2016
Accepted 10 July 2016

Keywords:
Series solutions
Lamb's solution
Wave functions
Elastic waves
Scattering problem

ABSTRACT

This paper presents a semi-analytical method for studying the two-dimensional problem of elastic wave scattering by surface irregularities in a half-space. The new method makes use of the member of a c-completeness family of wave functions to construct the scattering fields, and then applies equal but opposite tractions to those of the foregoing constructed scattering fields on the horizontal surface of the half-space to produce additional scattering fields. These additional scattering fields are a series of Lamb's solutions. Thus the whole scattering field constructed in the series automatically satisfies the Navier equations, the condition of zero traction on the half-space surface, and the radiation boundary conditions at infinity. Using the traction-free conditions along the canyon surface, the coefficients of the series solutions are determined via a least-squares method. For incident P, SV, and Rayleigh waves, the numerical results are presented for the scattering displacements in the vicinity of a semi-circular canyon in the half-space.

1. Introduction

It is well known that wave scattering may induce large amplifications and variations in ground motion during earthquakes [1,2]. As this effect is an important factor for making decisions in seismic risk estimation and for calculating structural responses, it is necessary to make more accurate predictions of ground amplifications for both seismologists and earthquake engineers [3,4].

Due to the involvement of infinite domains of wave-scattering problems, it is difficult to solve these problems using the conventional numerical methods [5–7]. To avoid any reflections from an artificially truncated boundary, a large computational domain is needed, thus large calculation loads are inevitable. Compared to numerical methods, analytical or semi-analytical methods require less computational effort, and the results are not influenced by the selection of the computational domain and grid divisions.

In particular, analytical or semi-analytical solutions usually play a key role in understanding the fundamental behavior for many different types of scientific and engineering problems. For this reason, great efforts have been made to obtain analytical or semi-analytical solutions over the past decades. For instance, analytical or semi-analytical solutions have been derived for the following wide range of problems: (1) wave propagation in the dry and water-saturated half-space [8–11]; (2) the convective pore-fluid flow problem in a water-saturated half space [12–15]; (3) the physical dissolution-front instability problem in water-saturated infinite domains [16,17]; (4) the chemical dissolution-front instability problem in a water-saturated full space [18–23]; (5) the pore-fluid focusing problem within a crack in a water-saturated full space [24–26]. Despite the above-mentioned important roles, analytical solutions are often difficult to be derived because the involvement of complicated and strict mathematical deductions [8–26], particularly when compared with obtaining approximate solutions using numerical methods. Therefore, it remains desirable to develop analytical and semi-analytical methods for dealing with elastic wave scattering problems along a canyon in a half space. There is no doubt that the outcome of this research can find certain applications in the emerging computational geoscience field [27].

After Trifunac [10], who studied two-dimensional elastic wave scattering analytically, many researchers have investigated wave scattering problems along irregular surfaces in a half-space for various incident wave fields [2]. Being uncoupled with other body waves, SH-wave scattering is the simplest one. Hence, for an SH-wave incident upon a canyon or an alluvial valley with semi-circular or semi-elliptical shapes, the exact solution can be obtained by the separation of variables approach [11,28,29]. With regards to the incidence of P, SV, and Rayleigh waves, wave scattering solutions are much more complicated as the boundary conditions are coupled [30–33]. To solve this problem many techniques have been proposed. For small slope irregularities, the perturbation method and asymptotic expansions have been used [34]. Under

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http://dx.doi.org/10.1016/j.soildyn.2016.07.006

the assumption of the periodicity of surface boundaries, solutions have been obtained for different wave fields and scatterings [35]. By modifying the half-space horizontal boundary as an almost flat circular boundary, Cao and Lee [36] studied the scattering and diffraction of plane P waves by cylindrical canyons. However, the condition of zero traction on the free-surface of the half-space is met only approximately [3].

The fictitious source method has been widely applied to solve cases of incident P, SV, and Rayleigh waves at canyon-like topographies [30,37]. By making use of the Lamb solutions, this method uses fictitious discrete sources buried outside the region. The fictitious source method rigorously satisfies both the free surface conditions on the half-space surface and the Sommerfeld radiation conditions at infinity, while approximately satisfying the boundary conditions on the surface of the irregularity in the least-squares sense. In addition, the location of fictitious sources and their number requires great care [32].

Of all the available boundary methods, Trefftz's method is the most efficient. It constructs scattering fields with a c-completeness family of solutions for wave equations, instead of using Lamb's solutions [38]. Since each member of this family does not satisfy the free-boundary conditions on the half-space surface, the matching at the boundary should also include a free plane surface. For obtaining convergent results, the length of the considered free boundary approximately requires two or three times the characteristic dimension of the scattering body [32]. Moreover, the constructed scattering fields do not include the diffracted Rayleigh waves, with some additional errors thus being introduced.

In this paper, the two-dimensional elastic wave scattering problem along a surface irregularity in the half-space is expressed in the frequency domain, including the governing equation and boundary conditions. Then a series solution method for the problem is proposed. Finally, for incident P, SV, and Rayleigh waves, some numerical results are first presented for the displacement responses along a semi-circular canyon in the half-space and then compared with existing results to verify their effectiveness.

2. Problem statement

By considering an elastic half-space with a two-dimensional surface irregularity, as shown in Fig. 1, two coordinate systems of the same origin are employed: a rectangular coordinate system, and a cylindrical coordinate system. The origin should be set at the point on the extension of the half-space surface above the irregularity. These coordinate systems will be used in the derivation of the proposed method in the next part of this paper. The medium of the half-space is assumed to be linearly elastic, homogeneous and isotropic. Material properties are specified by Lamé constants λ and μ, and the mass density ρ. The half-space experiences the incidence of P, SV, and Rayleigh waves. For the harmonic dependence of the type $\exp(i\omega t)$, the reduced Navier equation [39] for the half-space is:

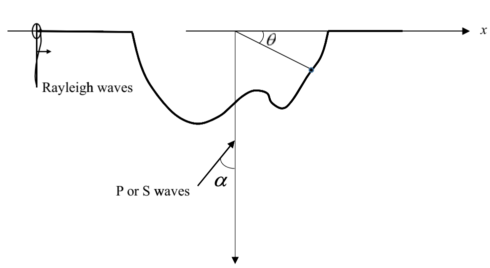

Fig. 1. An irregular canyon in half-space that is subjected to incident P, SV, and Rayleigh waves.

$$\mu\nabla^2\mathbf{u} + (\lambda+\mu)\nabla\nabla\cdot\mathbf{u} + \rho\omega^2\mathbf{u} = \mathbf{0} \tag{1}$$

where $\nabla^2=\partial^2/\partial x^2+\frac{\partial^2}{\partial z^2}$ is the Laplace operator in rectangular coordinate form; $\boldsymbol{u}=[u \;\; w]^T$ is the displacement vector, and ω is the circular frequency.

Moreover, the boundary conditions have zero traction along both the infinite horizontal surface of the half-space and the surface of the irregularity.

3. Solution method

To analytically solve the foregoing scattering problem, we introduce the potential functions ϕ and ψ as follows [40]:

$$u=\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial z} \tag{2a}$$

$$w=\frac{\partial\phi}{\partial z}+\frac{\partial\psi}{\partial x} \tag{2b}$$

The reduced Navier Eq. (1) will be satisfied if these two potentials are solutions of the following corresponding wave equations:

$$(\nabla^2+p^2)\phi(x,z)=0 \tag{3a}$$

$$(\nabla^2+s^2)\psi(x,z)=0 \tag{3b}$$

where $p=\omega/c_p$ is the wavenumber of the P wave, and $s=\omega/c_s$ is the wavenumber of the S wave. Moreover, c_p and c_s denote the longitudinal and transverse wave velocities, respectively.

For a half-space with a surface irregularity under incidence of elastic waves, as shown in Fig. 2(a), the total wave field can be decomposed as a free field (as shown in Fig. 2(b)) and a scattering field. The total wave field consisting of the free field and the scattered field are expressed as:

$$\phi^T=\phi^I+\phi^S \tag{4a}$$

$$\psi^T=\psi^I+\psi^S \tag{4b}$$

where the superscripts T, I, and S stand for the total, incidence, and scattering fields, respectively. The tractions σ^f and τ^f in Fig. 2(b) are the normal stress and the shear stress on the surface of the irregularity in the free field, respectively. The two stresses make up the surface stress vector $\mathbf{t}_f$, which can be obtained from the stress tensor of the free field $\boldsymbol{\sigma}_f$ and the normal vector of the surface $\mathbf{n}_s$:

$$\mathbf{t}_f=\boldsymbol{\sigma}_f\cdot\mathbf{n}_s \tag{5}$$

The stress tensor of the free field, $\boldsymbol{\sigma}_f$, can be obtained from the potential functions of the P, SV or Rayleigh waves. The expressions of the potential functions of different waves are in the Appendix A.

Furthermore, the scattering field consists of two parts. One part is constructed with the c-completeness set of solutions for the wave equations, but each member of this set does not satisfy in itself the zero traction condition on the half-space surface, as shown in Fig. 2(c). The tractions $\sigma_{\theta\theta}^n$ and $\sigma_{r\theta}^n$ are the normal stress and the shear stress produced by the nth-order solution of wave functions on the half-space surface, respectively, and the tractions σ^n and τ^n are the normal stress and the shear stress produced by the nth-order solution of wave functions on the surface of the irregularity, respectively. To satisfy the zero traction condition on the half-space surface, equal but opposite tractions $-\sigma_{\theta\theta}^n$ and $-\sigma_{r\theta}^n$