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Wave motion equation and the dynamic Green's function for a transverse isotropic multilayered half-space

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Abstract

A new formulation is presented here for harmonic wave motion in a transverse isotropic multilayered half-space. By means of a Fourier-Bessel transform, the complex partial differential equations of wave motion can be uncoupled into a pair of second order ordinary differential equations: one for SV-P vectorial waves (matrix size 2×2) and the other for SH scalar waves (matrix size 1×1). They have the same form as that for isotropic media. Thus, the same solution procedure as that for isotropic media is equally applicable to transverse isotropic media, which considerably simplifies the solution. Furthermore, by introducing a mixed variable formulation of the wave motion solution, the matrix form of Green's function for various boundary conditions of stratified soil is analytically derived. Numerical examples of Green's function and the dynamic foundation impedance demonstrate the accuracy and the efficiency of the proposed approach. The computation is unconditionally stable.

Keywords: Wave motion; Transverse isotropic medium; Multilayered half-space; Green's function; Dynamic impedance of embedded foundation

1. Introduction

The study of wave motion and Green's function is a subject of fundamental interest within the context of assessing the seismic safety and vulnerability of large and complex infrastructures because of its relevance to soil-structure interaction, geotechnical earthquake engineering, foundation vibration and seismology. It has been evidenced by numerous researchers, such as Ward et al. (1965), Pickering (1970) and Arthur and Menzies (1972), that soils in nature invariably exhibit some degree of anisotropy in their response to static and dynamic loads. Since the inception of the well-known work by Wolf (1935), extensive research has been carried out with regard to wave motion in anisotropic media, such as works contributed by Bardeb (1963), Barnett (1972), Beskos (1997), Wang and Denda (2007) and Tonon et al. (2001). To date, however, not many papers have been devoted to the problem of the dynamic Green's function in transverse isotropic and anisotropic multilayered half-spaces. Eskandari-Ghadi (2005) and Rahirmian et al. (2007) proposed an elastodynamic potential method to deal with transverse isotropic media; Khojasteh et al. (2011) presented a solution for 3D Green's functions for a multilayered transverse isotropic half-space with the aid of the potential method; and Amiri-Hezaveh et al. (2013) presented a solution for vertical and horizontal impedance functions for a surface rigid rectangular foundation on a transverse isotropic multilayered half-space using the potential method and the Hankel transform. Kausel and Roesset (1981) and Oliveira Barbosa and Kausel (2012) proposed the stiffness matrix method and generalized the thin-layer method to deal with

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wave propagation and Green's function in a 3D cross-anisotropic space. The authors of this paper have presented a solution for the dynamic impedance functions of an arbitrary-shaped rigid foundation on an anisotropic multi-layered half-space (Lin et al., 2014, 2015) by employing a multi-Fourier transform approach.

This paper extends and improves on a previous work. The contribution of the present work lies in the following issues. Firstly, a new formulation of the wave motion equation is proposed. By employing the Fourier-Bessel transform with special manipulation, the complex partial differential equations in the wave-number domain can be successfully reduced to a pair of uncoupled ordinary differential equations of the second order of matrix sizes 2×2 and 1×1: one for vectorial SV-P waves and the other for scalar SH waves. They have the same analytical solutions and present the same form as that for isotropic media. Thus, the same solution procedure as that used for isotropic media can be equally applied to transverse isotropic media, which results in greatly simplifying the solution. Furthermore, a mixed variable formulation of the wave motion solution is proposed; this enables a simple and convenient evaluation of Green's function. As a result, the matrix form of Green's function under various boundary conditions of stratified soil, underlain by an elastic half-space, is derived. A comparison with the numerical solutions for Green's function in the interior of a transverse isotropic multilayered strata, available in the literature, confirms the accuracy of the proposed approach. Then, the effect of material anisotropy on Green's function and the dynamic impedance of foundations embedded in a multilayered half-space is studied.

2. Statement of the problem

A transverse isotropic multilayered half-space is considered (Fig. 1). It is assumed that the system is of the horizontally layered type and the vertical axis is chosen to be the axis of radial symmetry. The equations of motion for

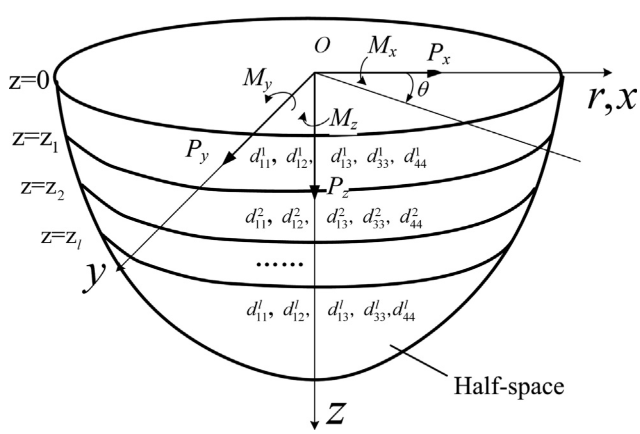

Fig. 1. Multilayered transverse isotropic half-space.

a layer in the cylindrical coordinates, with the z axis pointing downwards, are expressed as

$$\begin{aligned} &\frac{\partial \sigma_r}{\partial r}+\frac{1}{r}\frac{\partial \tau_{\theta r}}{\partial \theta}+\frac{\partial \tau_{zr}}{\partial z}+\frac{\sigma_r-\sigma_\theta}{r}=\rho\frac{\partial^2 u_r}{\partial t^2}\\ &\frac{\partial \tau_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta}+\frac{\partial \tau_{z\theta}}{\partial z}+\frac{2\tau_{r\theta}}{r}=\rho\frac{\partial^2 u_\theta}{\partial t^2}\\ &\frac{\partial \tau_{rz}}{\partial r}+\frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta}+\frac{\partial \sigma_z}{\partial z}+\frac{\tau_{rz}}{r}=\rho\frac{\partial^2 u_z}{\partial t^2} \end{aligned} \tag{1}$$

where σ_r, σ_θ and σ_z are the normal stress components in the cylindrical coordinates in the r, θ and z directions, respectively; $\tau_{\theta r}$, τ_{zr} and $\tau_{z\theta}$ are the corresponding shear stress components and u_r, u_θ and u_z are the corresponding displacement components, respectively; and ρ is the mass density.

The constitutive equations for transverse isotropic media are expressed as

$$\begin{aligned} &\sigma_r = d_{11}\varepsilon_r + d_{12}\varepsilon_\theta + d_{13}\varepsilon_z\\ &\sigma_\theta = d_{12}\varepsilon_r + d_{11}\varepsilon_\theta + d_{13}\varepsilon_z\\ &\sigma_z = d_{13}\varepsilon_r + d_{13}\varepsilon_\theta + d_{33}\varepsilon_z \end{aligned} \tag{2}$$

$$\tau_{\theta z} = d_{44}\gamma_{\theta z}, \quad \tau_{zr} = d_{44}\gamma_{zr}, \quad \tau_{r\theta} = d_{66}\gamma_{r\theta} \tag{3}$$

where $d_{66} = (d_{11} - d_{12})/2$. A transverse isotropic elastic medium is characterized by five independent constants: Young's modulus in the plane of isotropy, E_{hh}, and perpendicular to it, E_{hv}, the shear modulus in a plane perpendicular to the plane of isotropy, G_{hv}, and Poisson's ratio in the plane of isotropy, v_{hh}, and perpendicular to it, v_{hv}. Elastic constants d_{11}, d_{12}, d_{13}, d_{33} and d_{44} are related to Young's moduli and Poisson's ratios as follows:

$$\begin{aligned} &d_{11} = (E_{hh}/a)(1 - nv_{hv}^2)\\ &d_{12} = (E_{hh}/a)(nv_{hv}^2 + v_{hh})\\ &d_{13} = (E_{hh}/a)v_{hv}(1 + v_{hh})\\ &d_{33} = (E_{hv}/a)(1 - v_{hh}^2)\\ &d_{44} = G_{hv} \end{aligned} \tag{4}$$

in which

$$\begin{aligned} &n = E_{hh}/E_{hv}\\ &a = (1 + v_{hh})(1 - v_{hv} - 2nv_{hv}^2) \end{aligned} \tag{5}$$

Thermodynamic consideration requires that the strain energy of an elastic material should always be positive. This imposes certain restrictions on the acceptable range of elastic constants.

$$\begin{aligned} &d_{66} > 0, \quad d_{44} > 0, \quad d_{11} - d_{66} > 0,\\ &(d_{11} - d_{66})d_{33} > d_{13}^2 \end{aligned} \tag{6}$$

For isotropic media, we have $E = E_{hh} = E_{hv}$ and $v = v_{hh} = v_{hv}$. This results in $d_{11} = d_{33} = \lambda + 2G$, $d_{12} = d_{13} = \lambda$ and $d_{44} = d_{66} = G$ with λ and G being Lame's constants. The corresponding material constants are then reduced to two.

The strain-displacement relationship is expressed as