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Quantifying the cross-correlation between effective cohesion and friction angle of soil from limited site-specific data

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Abstract

The effective cohesion (c') and effective friction angle (ϕ') of soil are important soil parameters required for evaluating stability and deformation of geotechnical structures. It is well known that there is cross-correlation between c' and ϕ' of soil and that this cross-correlation affects reliability analysis of geotechnical structures. Ignoring the cross-correlation between c' and ϕ' may lead to a biased estimation of failure probability. It is therefore important to properly quantify the cross-correlation between c' and ϕ' of soil for geotechnical analysis and design. However, the c' and ϕ' data obtained from field and/or laboratory tests for a project are usually limited and insufficient to provide a meaningful joint probability distribution of c' and ϕ' or quantify their cross-correlation. This poses a significant challenge in engineering practice. To address this challenge, this paper develops a Bayesian approach for characterizing the site-specific joint probability distribution of c' and ϕ' and quantifying the cross-correlation between c' and ϕ' from a limited number of c' and ϕ' data obtained from a project. Under a Bayesian framework, the proposed approach probabilistically integrates the limited site-specific c' and ϕ' data pairs with prior knowledge, and the integrated knowledge is transformed into a large number of c' and ϕ' sample pairs using Markov Chain Monte Carlo (MCMC) simulation. Using the generated c' and ϕ' sample pairs, the correlation coefficient of c' and ϕ' is estimated, and the marginal and joint distributions of c' and ϕ' are evaluated. The proposed approach is illustrated and validated using real c' and ϕ' data pairs obtained from direct shear tests of alluvial fine-grained soils at Paglia River alluvial plain in Central Italy.

Keywords: Effective cohesion; Effective friction angle; Bayesian approach; Markov Chain Monte Carlo simulation; Joint probability distribution; Correlation coefficient

1. Introduction

Determination of shear strength parameters (e.g., effective cohesion, c', and effective friction angle, ϕ') of soil is vital for evaluating stability and deformation of geotechnical structures such as foundations, slopes and retaining walls. Previous studies have explored and evaluated performance of shallow foundations (e.g., Cherubini, 2000), embankments (e.g., Matsuo and Kuroda, 1974; Hata et al., 2008), earth dams (e.g., Babu et al., 2007; Babu and Srivastava, 2010) and slopes (e.g., Wolff, 1985; Tang, et al., 2013) using c' and ϕ' data as input. The c' and ϕ' are the parameters (i.e., intercept and slope respectively) of a failure envelope of soils. The Mohr-Coulomb failure criterion (e.g., Gan et al., 1988) is commonly used to model this failure envelope by fitting it to a given number of shear strength observation data obtained from field or laboratory tests such as triaxial or direct shear tests. During

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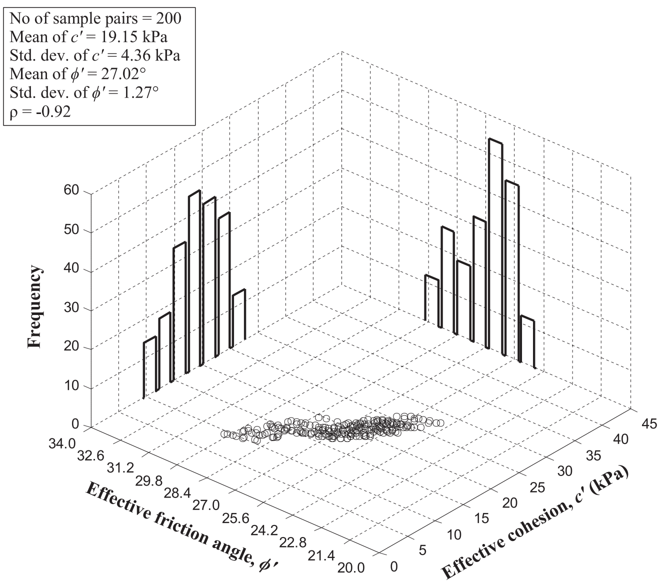

Fig. 1. Effective cohesion and friction angle measured by direct shear tests at Paglia River alluvial plain in Central Italy (data from Di Matteo et al. (2013)).

the fitting, if the slope (i.e., ϕ') is overestimated, then the intercept (i.e., c') of the fitted line tends to be underestimated, and vice versa. Therefore, c' and ϕ' are not independent of each other, but possess some degree of cross-correlation between them.

Cross-correlation between two soil parameters (e.g., c' and ϕ') in this study refers to a relationship between the soil parameters such that a variation in the values of one parameter (e.g., c') leads to variation in the values of the other parameter (e.g., ϕ') (e.g., Fenton and Griffiths, 2003; Wu, 2013; Le, 2014). The cross-correlation between c' and ϕ' has been studied previously, and a negative cross-correlation between c' and ϕ' has been widely reported (e.g., Lumb, 1970; Yucemen et al., 1973; Wolff, 1985; Young, 1986; Cherubini, 1997; Hata et al., 2008; Forrest and Orr, 2010; Rahardjo et al., 2012; Di Matteo et al., 2013). For example, Di Matteo et al. (2013) obtained a value of -0.92 for the cross-correlation between the c' and ϕ' values of alluvial fine-grained soils, as illustrated in Fig. 1.

In a reliability analysis of geotechnical structures, the cross-correlation between c' and ϕ' plays an important role in the determination of failure probability. Ignoring the cross-correlation in a probabilistic analysis may lead to the underestimation or overestimation of the failure probability (e.g., Baecher and Christian, 2003; Low, 2007; Li et al., 2011, 2012; Tang et al., 2012, 2013; Wu, 2013; Jiang et al., 2014a). Therefore, the failure probability may be severely biased when using an assumption of independence (i.e., zero cross-correlation) between c' and ϕ' (e.g., Jiang et al., 2014b). Previous studies have shown that the probability of failure is sensitive and highly dependent on the correlation coefficient between c' and ϕ' (e.g., Wolff, 1985; Cherubini, 2000; Soubra and Mao, 2012; Tang et al., 2013; Wu, 2013, Jiang et al., 2014b). In addition, the evaluation of the reliability of geotechnical structures may require joint probability distributions of correlated geotechnical parameters. For example, Tang et al. (2013) and Li et al. (2015) mentioned that the joint probability distributions of correlated geotechnical parameters must be known in order to evaluate the exact reliability of geotechnical structures. Copula-based approaches have recently been applied in geotechnical engineering to simulate the joint probability distribution of c' and ϕ' (e.g., Tang et al., 2013; Wu, 2013), but such analyses require the cross-correlation between c' and ϕ' as input. This underscores the importance of determining a meaningful site-specific joint probability distribution and cross-correlation between c' and ϕ' for geotechnical probabilistic analyses.

The c' and ϕ' of soils are frequently determined simultaneously from field and/or laboratory tests (e.g., borehole shear, direct shear and triaxial tests). However, the number of c' and ϕ' data pairs obtained from these tests is usually limited because the field or laboratory tests require significant commitment of time, resources and finance. Note that conventional statistical methods require a minimum number of data for using their equations to calculate standard deviation or correlation coefficient (e.g., Walpole et al., 1998). For example, a “rule of thumb” minimum number of data required for the standard deviation or correlation coefficient equation is 30 (e.g., Walpole et al., 1998). If the number of data is too small (i.e., much smaller than 30), the conventional statistical methods provide significantly biased statistics. For example, if only one data point is sampled from a population, direct application of the standard deviation equation to one data point leads to a standard deviation value of zero. Similarly, when only two data pairs of two correlated parameters are sampled from a population, direct application of the correlation coefficient equation to these two data pairs results in a perfect correlation (i.e., correlation coefficient being ± 1). These statistical estimates obtained are obviously biased. Although standard deviation or correlation coefficient can be calculated from limited data points using conventional statistical equations, the values obtained are not meaningful.

In geotechnical engineering practice, the number of c' and ϕ' data pairs obtained from field or laboratory tests in a specific site is often smaller than 30, particularly for projects with relatively small or medium size (e.g., Wolff, 1985; Wang and Cao, 2013). It is therefore difficult to obtain meaningful statistics (e.g., mean and standard deviation) for c' and ϕ', respectively, and it is even more difficult to estimate the cross-correlation between c' and ϕ' from limited data pairs for a specific site. A Bayesian equivalent sample method has recently been developed to obtain meaningful statistics of a single geotechnical parameter from limited data (Wang and Cao, 2013; Cao and Wang, 2014). This study aims to further develop the Bayesian equivalent sample method to properly quantify the joint probability distribution and cross-correlation of two geotechnical parameters (e.g., c' and ϕ').

The paper starts with probabilistic modeling of uncertainty in c' and ϕ', followed by the development of the joint probability density function (PDF) of c' and ϕ' using a Bayesian framework. The derived joint PDF of c' and ϕ' is then incorporated into Markov Chain Monte Carlo (MCMC) simulation to generate a large number of c' and ϕ' sample pairs. The large number of sample pairs generated by MCMC