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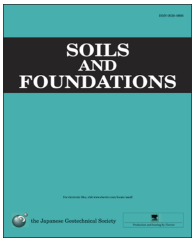

Technical Paper

Numerical analyses of steady-state seepage problems using the interpolation finite difference method

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Abstract

Seepage analyses have mainly been executed using the finite element method; numerical analyses using the finite difference method (FDM) have been limited to cases where the calculation domains are comparatively simple. This limitation is observed because FDM is considered to be inappropriate for application in calculations over complex domains. However, by applying the so-called "interpolation FDM (IFDM)", we can now freely solve two- and three-dimensional elliptic partial differential equations (PDEs) over complex domains with high speed and high accuracy. By adopting this procedure, named the boundary polynomial interpolation, all of the numerical analyses of elliptic PDEs reduce to Dirichlet problems over regular domains. This method is also effective in the calculation of a flow net where mixed Dirichlet and Neumann conditions exist. By giving the coordinate values of changing points regarding the polygonal line of a domain and boundary conditions, grid generation is automatically carried out and numerical solutions are promptly obtained. In this paper, the method of saturated seepage analyses with a fixed domain is first formulated and then expanded to unconfined domain problems, namely, free surface problems. While analytical solutions of the PDE are highly limited, there is an analytical solution for the location of the free surface in a rectangular dam. The numerical solutions obtained using the IFDM are compared with the analytical ones, and it is shown that the proposed method has adequate accuracy in practice and wide applicability as a general method of numerically solving seepage problems.

Keywords: Seepage analysis; Interpolation FDM; Arbitrary domain; Flow net; Free surface

1. Introduction

Under the condition that hydraulic transmissivity is both homogeneous and isotropic, the governing equation of steady seepage problems becomes the Laplace equation. The Laplace equation is also the governing equation in the study of steady state thermal fields, electrostatic fields, and potential flows, etc. All of the phenomena over the fields are commonly clarified by solving the Laplace equation over each of the domains and boundary conditions. However, in the seepage problems of geotechnical engineering, the coefficient of permeability is usually anisotropic, and the governing equation therefore changes from the Laplace equation to an elliptic PDE. Moreover, there are problems of saturated and unsaturated infiltrations; finally, we may have to simulate the seepage flow as a transient problem. These problems have been studied by many researchers, and commercial software applicable to such problems is available, as described later. In this paper, a new idea is introduced to the field of the seepage-flow problem; therefore, we will provide a brief history of this topic.

In the era when computational devices were not adequate, the free surface line and discharge of an earth dam were determined using a graphical technique. Among the classical approaches to the seepage problem, the method proposed by Casagrande (1937) is an exemplary one. He made up flow nets of some typical type earth dams, obtained the whole configuration of steady state infiltration, and then proposed a method to estimate the free surface line and discharge in a simple manner. His method was adopted in Japan and used as a standard in the design of homogeneous earth dams (ASIB, 1981).

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Nomenclature

C_i	convergence factor, $=1–10^{-m}$
d_f	deletion factor
$E_{i,j}$	calculation element
E_{i_c,j_c}	convergence judgement element
h	total head
$l_{wi,j}$	wall distance factor
m	convergence index
n_e	final calculation step
N_x	x direction division number
N_y	y direction division number
p	pressure head
q_C	calculated discharge after Casagrande
q_n	numerically calculated discharge
s	stream function
u	x direction velocity
v	y direction velocity
x	horizontal direction coordinate
y	vertical direction coordinate
α_a	acceleration factor of FTCS scheme
$\alpha_{a\max}$	analytical MAF
α_b	acceleration factor of TMSD scheme
$\alpha_{b\max}$	theoretical MAF of TMSD scheme
α_{bopt}	OAF of TMSD scheme
$\alpha_{n\max}$	numerical MAF of TMSD scheme
α_{nopt}	numerical OAF as non-decreasing solution
α'_{nopt}	numerical OAF as allowable-oscillation solution
Δx	x direction difference
Δy	y direction difference
Δt	time difference
Λ	allowable- oscillation factor

During and after World War II, various methods of numerical analysis for a continuum body were rapidly developed, corresponding to the uprising of computer performance and its popularization. The finite difference method (FDM) (Shortley and Weller, 1938; Shaw and Southwell, 1941; Freeze, 1971a, 1971b; Aitchison, 1972; Mahmud, 1996; Bardet and Tobita, 1996; Borja and Kishnani, 1991, Harbaugh, 2005), the finite volume method (FVM) (Darbandi et.al. 2007), the finite element method (FEM) (Westbrook, 1985; Lam et al., 1987; Thieu et al., 2001; GEO-SLOPE International Ltd, 2009), and the boundary element method (BEM) (Leontieva and Huacasib, 2001; Chen et al., 2007; Chantasiriwan, 2011) were developed and applied to various engineering problems. The above-referenced literature is limited to those of the seepage analyses referred to in this paper. There are other methods which employ adaptive (curvilinear) coordinate systems in the field of FDM, which are beyond the scope of the paper.

Here, let us briefly describe these methods, noting their key attributes as found in the literature (Ferziger and Perić, 2002). FDM is the oldest method for numerical solutions of PDEs. On structured grids, the FDM is very simple and effective. It makes it especially easy to obtain higher-order schemes on regular grids. The disadvantage of the FDM is that certain conservation laws are not enforced unless special care is taken. Furthermore, the restriction to simple geometries is a significant disadvantage if complex flows are specified. The FVM can accommodate any type of grid, so it is suitable for complex geometries. The disadvantage of FVMs compared with FD schemes is that methods higher than second order are more difficult to develop in 3D. The FEM has an important advantage in being able to handle arbitrary geometries. The principal drawback, which is shared by any method that uses unstructured grids, is that the matrices of the linearized equation are not as well-structured as those for regular grids, making it more difficult to find efficient solution methods. In the FVM and FEM, because unstructured grid systems are used, grid points are generated somewhat arbitrarily, and a considerable amount of preprocessing time is required. To overcome this drawback, the BEM and some meshless methods (Takbiri et al., 2010; Chaiyo et al., 2011) have been proposed. Because there are many numerical methods, we can effectively choose a suitable one from a wide range of methods. Among them, however, the FEM is the most popular and widely used. In the field of seepage problems, a general-purpose system, e.g., SEEP/W software (GEO-SLOPE International Ltd, 2007), was published and was enthusiastically welcomed: “these analytical methods have now moved from being research tools to application tools. This has opened a whole new world of numerical modeling.”

In this paper, the FDM is used to numerically analyze seepage problems. We make a survey of the literature concerned with the numerical calculations of seepage problems using the FDM. The FDM discretization of the Laplace equation was given by Shortley and Weller (1938). Their scheme, the Shortley-Weller scheme, is a second-order accurate scheme, and the grid width can be designated arbitrarily; that is, it can be applied to the calculation over any irregular domains. Shaw and Southwell (1941) investigated some free surface problems using a relaxation method. In their method, a pressure head is adopted as the calculation variable. While the figurative expression of iso-pressure-head lines seems to be valid, the method involves no distinct numerical data of the free surface line regarding the rectangular dam. Aitchison (1972) used the FDM to determine the position of the free surface of a rectangular dam. “Based on the semi-analytical nature of Aitchison's solution by using the complex variable, his data are more believable than other numerical results (Chen et al., 2007).” The discretization of the governing equation using the FDM can be linked to an electronic spreadsheet, such as Microsoft Excel; in particular, if the two-dimensional calculation domain is a regular one, we can execute an iterative calculation only by defining a pertinent equation for each of the cells to obtain an adequately accurate numerical solution. Using a spreadsheet, Mahmud (1996) executed the calculation regarding the seepages through the foundations of gravity dams. Bardet and Tobita (2002) investigated the calculation method regarding the free surface seepage problems of rectangular and trapezoidal dams. In their method, a pressure head is adopted as the calculation variable. The essential concept of their method is the extended pressure method (Borja and Kishnani, 1991), and the location of the free surface is obtained as the result of convergence. “The proposed method (using the spread sheet) has not only educational values because it openly describes the equations used in solving free-surface seepage problems but also practical values because it is applicable to many free-surface seepage problems.” The spreadsheet calculation is indigenous to the FDM and is not applied to the other methods. One drawback of this method is that we have to make the spreadsheet separately. Furthermore, in the calculation of an irregular domain, calculation accuracy cannot be guaranteed.