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An efficient sparse Bayesian learning framework for stochastic response analysis

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ABSTRACT

The computational intensiveness inherently associated with uncertainty quantification of engineering systems has been one of the prime concerns over the years. In order to mitigate this issue, a novel approach has been developed for efficient stochastic computations. The proposed approach has been developed by amalgamating the advantages of two available techniques namely, high dimensional model representation (HDMR) and Kriging. These two methods are coupled in such a way that HDMR addresses the global variation in the functional space using a set of component functions and the fine aberrations are interpolated by utilizing Kriging, performing as a two level approximation. A Bayesian learning framework has been integrated with the locally refined model so as to construct a sparse configuration. Implementation of the proposed approach has been demonstrated with five benchmark problems and a practical offshore structural problem. The efficiency and accuracy of the proposed approach in stochastic response analysis have been assessed by comparison with Monte Carlo simulation. Excellent results in terms of accuracy and computational effort obtained makes the proposed methodology potential for further complex applications.

1. Introduction

The integration of computational models and probabilistic methodologies have acquired notable interest in the last few decades. Consequently, the field of uncertainty quantification (UQ) has emerged quite rapidly and various approaches have been developed for accounting uncertainties in the structural models [1,2]. However, most of the approaches associated with UQ rely upon repeated calls to the underlying computational model of the structure [3,4]. More specifically, despite considerable advances in computer technology over the last two decades, a single simulation of large-scale finite element models still remain computationally expensive due to the continuous requirement for more realistic representation of the actual system response [5,6].

In order to attenuate the computational expense inherently associated with UQ, the concept of surrogate modelling has emerged [7,8]. Surrogate modelling is an efficient apparatus for formulating an algebraic approximation to the input-response map of the system. These tools approximate the underlying computational model in a sample space and thereby diminishing the simulation time appreciably [9]. Few noteworthy examples of such techniques which have been favorably employed in the past are, least square approximation [10], moving least square approximation [11], polynomial chaos expansion [12], high dimensional model representation (HDMR) [13], Kriging [14], radial basis function [15], support vector machine [16].

Among these techniques, HDMR has evolved to be one of the most conducive dimension reduction methods and has received prominent recognition in recent times [17]. It can be defined as a quantitative evaluation and estimation tool for capturing the high dimensional relationships between sets of input and output model variables. HDMR can be broadly categorized into two divisions on the basis of determining the component functions, which are cut-HDMR [18] and random sampling (RS)-HDMR [19]. However, these standard HDMR models constructed over the whole functional space has been often observed to be deficient in accurately capturing the input-output relationship [20,21]. Additionally, the computational effort associated with cut-HDMR may prove to be intensive in high dimensional space, since it utilizes interpolation [22,23]. The computational issue of interpolation can be surpassed by utilizing RS-HDMR approach. However, the determination of the component functions by Monte Carlo integration generally requires a large number of training points and moreover, introduces significant errors [24]. Therefore, the need to develop

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efficient HDMR models operable in high-dimensional functional space has been the mainstay among the research community. In this context, a new multiple sub-domain RS-HDMR has been proposed [25], which is a combination of RS-HDMR approximations built in sub-domains of input space. Very recently, a similar approach referred to as multi-element least square HDMR has been proposed [26], in which various least square regression techniques have been utilized for stochastic multiscale model reduction.

In most applications, it has been observed that the responses are sparse and contain few important terms in their sphere. Efficient approaches have been developed for reconstructing sparse functions in the field of compressed sensing [27]. This reconstruction problem is an ill-posed problem and regularization techniques (e.g., Tikhonov regularization) that constrain the ℓ_2 – norm of the solution are commonly employed [28]. An adaptive non-intrusive method has been proposed which builds a sparse PC expansion utilizing the least angle regression algorithm for detecting the significant coefficients of the PC expansion [29]. In the same context, a prevalent supervised machine learning algorithm referred to as support vector machine (SVM) [30] has been developed. SVM has been observed to lead towards good generalization by avoiding over-fitting and effectively results in a sparse model dependent only on a subset of kernel functions [31]. However, in order to address some of the shortcomings of SVM (which have been later discussed in Section 2), a sparse Bayesian framework identical in functional form to SVM, known as relevance vector machine (RVM) has emerged [32].

The objective of this work has been to develop an efficient computational model which is capable to deal with high-fidelity complex UQ problems by reviewing the state-of-the-art and addressing their limitations as discussed over the last two paragraphs. The primary focus has been to improvise the existing HDMR models in terms of prediction accuracy within limited computational budget. Specifically in the present work, the following improvements have been implemented on the HDMR model:

- As the first enhancement, Kriging has been incorporated within the HDMR model so as to achieve local refinements and ameliorate its approximation capabilities. A similar refinement to PCE [33] has been recently implemented which has served as a stimulus to this work.
- Since Bayesian formulation is considered to be the most general framework for UQ, a Bayesian framework of the above locally refined model has been devised by integrating an effective machine learning algorithm, referred to as RVM so as to construct a sparse model at a much lower computational complexity.

In this paper, the refined computational model proposed has been applied to the following cases:

- Five benchmark test problems for numerical validation,
- A practical offshore application, such as, stochastic response analysis of a four-legged offshore jacket platform.

The rest of the paper has been organized in the following sequence. Section 2 discusses the general foundation of the various approaches utilized in this study. The detailed formulation of the proposed approach has been presented in Section 3. The proposed approach has been applied to five benchmark problems in Section 4. Section 5 explores the performance of the proposed approach in a practical offshore structural problem. Finally, the useful findings of the study have been summarized.

2. Fundamental concepts

2.1. High dimensional model representation

In this sub-section, a brief description of HDMR as an effective tool to map the input-output functional relationship has been presented. Suppose, $\mathbf{i} = (i_1, i_2, \ldots, i_N) \in \mathbb{N}_0^N$ be a multi-index with $|\mathbf{i}| = i_1 + i_2 + \cdots + i_N$, and let $N \geqslant 0$ be an integer. Considering, $\mathbf{x} = (x_1, x_2, \ldots, x_N)$ be a N dimensional vector, representing the input variables of a structural system, the output $g(\mathbf{x})$ can be expressed as a finite series [34] as:

$$g(\mathbf{x}) = \sum_{|\mathbf{i}|=0}^{N} g_{\mathbf{i}}(\mathbf{x}_{\mathbf{i}}) \tag{1}$$

Definition 1. Two subspace A and H in Hilbert space are spanned by basis $\{a_1, a_2, \ldots, a_l\}$ and $\{h_1, h_2, \ldots, h_m\}$ respectively. If (i) $H \supset A$ and (ii) $H = A \oplus A^{\perp}$ where, $A^{\perp}$ is the orthogonal complement subspace of A in H, H is termed as extended basis and A as non-extended basis.

Considering ψ is to be a suitable basis of $\mathbf{x}$, Eq. (2) can be expressed in terms of extended bases as,

$$g(\mathbf{x}) = g_0 + \sum_{k=1}^{N} \left\{ \sum_{i_1=1}^{N-k+1} \cdots \sum_{i_k=i_{k-1}}^{N} \sum_{r=1}^{k} \left[\sum_{m_1=1}^{\infty} \sum_{m_2=1}^{\infty} \cdots \sum_{m_r=1}^{\infty} \alpha_{m_1 m_2 \ldots m_r}^{(i_1 i_2 \ldots i_k) i_r} \psi_{m_1}^{i_1} \ldots \psi_{m_r}^{i_r} \right] \right\} \tag{2}$$

where, g_0 is a constant term representing the zeroth order component function or the mean response of any response function $g(\mathbf{x})$. It has been observed that most real-life problems exhibit only the lower order cooperative effect and therefore, the higher order components in Eq. (2) can be ignored. Considering up to M^{th} order component function and s^{th} order basis, Eq. (2) can be rewritten as

$$\hat{g}(\mathbf{x}) = g_0 + \sum_{k=1}^{M} \left\{ \sum_{i_1=1}^{N-k+1} \cdots \sum_{i_k=i_{k-1}}^{N} \sum_{r=1}^{k} \left[\sum_{m_1=1}^{s} \sum_{m_2=1}^{s} \cdots \sum_{m_r=1}^{s} \alpha_{m_1 m_2 \ldots m_r}^{(i_1 i_2 \ldots i_k) i_r} \psi_{m_1}^{i_1} \ldots \psi_{m_r}^{i_r} \right] \right\} \tag{3}$$

Once the unknown coefficients associated with the bases are determined, Eq. (3) represents the basic functional form of HDMR. In order to obtain the coefficients $\boldsymbol{\alpha}$, a random sampling method and Monte Carlo integration are utilized. However, it has been observed that Monte Carlo integration experiences slow convergence and results to a significant error [19].

The following section introduces Kriging which is based on local functional approximation.

2.2. Kriging

Kriging has emerged as a powerful surrogate modelling technique in which the interpolated values are modelled by Gaussian process governed by prior covariances [35]. This key concept has been introduced [36] so that Kriging may also be utilized in simulation of computer experiments.

Suppose, $\mathbf{x} = (x_1, x_2, \ldots, x_N)$ are the input variables, where $\mathbf{x} \in D \subset \mathbb{R}_N$. Now assuming the model output $\mathbf{M}_{\mathbf{K}}(\mathbf{x})$ to be a realization of a Gaussian process, one obtains

$$\mathbf{M}_{\mathbf{K}}(\mathbf{x}) = \boldsymbol{\beta}^T \mathbf{f}(\mathbf{x}) + \sigma^2 \mathbf{Z}(\mathbf{x}, \boldsymbol{\omega}) \tag{4}$$