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Efficient response surface method for high-dimensional structural reliability analysis

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In common response surface method (RSM) for structural reliability analysis, performances of several experimental points must be evaluated via finite element analysis. The number of required experimental points is proportional to the number of random variables. Hence, for a high-dimensional structural reliability problem, computational cost is high, especially for structures with computationally intensive finite element models. On the other hand, the accuracy of classical RSM in estimating the probability of failure depends on the locations of experimental points. This paper proposes an efficient and accurate RSM. The efficiency is increased by using exponential surrogate model instead of quadratic one and by using experiment updating technique. In this way, the number of required experimental points is significantly reduced. Meanwhile, the accuracy of RSM is improved by choosing locations of experimental points judiciously, such that, to be close to the actual limit state surface. Five examples are solved to demonstrate the high efficiency and accuracy of the proposed method.

1. Introduction

The existence of uncertainty [1] in material properties, applied loads and geometrical characteristics of structures forms rational basis for structural reliability analysis. In order to assess the reliability of a structure, it is common to estimate its probability of failure. To this end, the use of numerical integration methods is unavoidable. This is because the closed-form solution for the integral of failure probability is not available in almost of all cases. One of the well-known simulation methods for estimating this probability is the Monte Carlo simulation (MCS) method [2], which needs very large number of evaluations of limit state function (LSF). Since each evaluation is accomplished through a finite element analysis, hence, the computational cost will be tremendous for large-scale structures with complex finite element models. On the other hand, first-order reliability method (FORM) [3,4] also requires a large computation time when a large number of random variables is involved. In addition, it may suffer from convergence problems [5]. In order to alleviate the computational burden of these time-consuming finite element analyses, it is convenient to use a response surface function (RSF) as a surrogate model instead of actual LSF. This well-known approach is called response surface method (RSM). In traditional RSM, quadratic polynomial function without cross terms is used as RSF [6]. The initial RSF is fitted based on $2n + 1$ experimental points located along the n axes of normal Gaussian space, where, n is the number of random variables. Then, FORM is applied to find the most probable point (MPP) for the initial RSF. Upon finding such a point (called also design point), the center point of the experimental points is replaced with this MPP and the other $2n$ points are selected around the MPP alongside the n axes (two points alongside each axis at both sides of MPP). This approach is repeated iteratively to find the final RSF. The efficiency of classical RSM depends on the number of random variables n, and on the number of iterations required to obtain final RSF. On the other hand, the accuracy of classical RSM in approximating actual LSF and consequently the probability of failure depends on the locations of experimental points around the design point.

Up to now, many researchers have improved the accuracy and the efficiency of classical RSM. These valuable works are briefly reviewed herein: Rajashekhar and Ellingwood [7] presented an adaptive iterative procedure to develop a RSF. They suggested a criterion for reduction in the number of experiments after the first iteration. The location of experiments for cross terms and a systematic procedure to reduce the number of experiments depending upon the importance of each term in the previous approximation also were suggested. Kim and Na [8] proposed an improved sequential RSM. They used gradient projection method to select sampling point close to actual LSF. They suggested also

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a method to control the selection range of sampling points, considering the nonlinearity of LSF. Gayton et al. [9] improved the RSM by taking into account the engineering knowledge about the structure under investigation. Their method, namely, CQ2RS (Complete Quadratic Response Surface with re-Sampling) reduces the cost of reliability analysis using a statistical formulation of the RSM. They used re-sampling of experiments and confidence interval to locate the design point. Gupta and Manohar [10] proposed an improved RSM useful in the case of reliability analysis involving performance functions with multiple design points and/or multiple failure regions with considerable contribution in the failure probability. Wong et al. [11] improved the RSM by choosing the parameter f as a decreasing function of the coefficient of variation of the random variables. An adaptive approach was also presented to modify the location of sampling points. In the adaptive RSM proposed by Kaymaz and McMahon [12], weighted regression was used in place of normal one to provide better approximation of LSF in vicinity of the design point. In addition, they selected experimental points from the region where the design point is most likely to exist. Duprat and Sellier [13] suggested to re-use the experimental points, which are positioned efficiently with respect to the design point, in the next iteration of the experimental design. Lee and Kwak [14] used moment method in combined with RSM to estimate the probability of failure more efficiently. The use of higher order polynomials in RSM can best be found in the work of Gavin and Yau [15]. They proposed to use a polynomial without fixed degree to obtain better RSF. Cheng et al. [16] presented an artificial neural network-(ANN) based RSM. They applied uniform design method to select training datasets for establishing an ANN model. Then, they used FORM to estimate the failure probability. Nguyen et al. [17] fitted the RSF using double weighted regression technique. In their method, experiments are weighted according to: (i) their distance from the true failure surface and (ii) their distance from the estimated design point. Doing so, computational time is reduced significantly. Kang et al. [18] proposed to use moving least squares method (MLSM) in RSM to find fitting constants. This increases the weights of points close to MPP, to provide better approximation of LSF around it. Also, the efficiency of RSM was improved in their work through judicious selection of experimental points in subsequent iterations. Allaix and Carbone [19] proposed an improvement of RSM. Their iterative strategy for determination of RSF, uses importance sensitivities of random variables to choose the locations of sample points. For the same number of LSF evaluations, the accuracy of their method in estimating failure probability is higher than classical RSM. Zhao and Qiu [20] proposed two improved RSMs. They reduced the number of LSF evaluations by using the concept of control point, which is close to the actual design point. In addition, a moving technique of experimental points was suggested to improve the accuracy of RSM in estimating the failure probability. Roussouly et al. [21] proposed a new adaptive RSM, in which a RSF is built from an initial Latin Hypercube Sampling (LHS) where the most significant terms are chosen from statistical criteria and cross-validation method. Then, LHS is refined in a stepwise manner and finally a bootstrap method is used to determine the influence of the response error on the estimated probability of failure.

In the above literature, the number of LSF calls is mainly reduced by reusing the experimental points of previous iterations. This is a good strategy for increasing the efficiency of RSM, but seems to be insufficient for the case of high-dimensional problems which has not been paid attention in the literature. As it was mentioned, in classical RSM, quadratic polynomial function without cross terms is used as RSF. In this approach, $2n + 1$ experimental designs are needed in each iteration, where, n is the number of random variables. Thus the total number of experimental points required for the construction of final RSF is proportional to the number of random variables. In most cases we need time-consuming finite element analyses to evaluate implicit performance function for experiments. In order to reduce the number of required experiments, in Section 3.2 of this paper, *exponential* response surface with only $n + 1$ sample points is used as RSF. In this way, the total number of experimental points will be reduced by about half for high-dimensional problems. Moreover, fitting procedure for exponential RSF will be converged faster than the same procedure for quadratic ones. This is because in this case, we will need to solve $n + 1$ equations for $n + 1$ unknowns. In addition to the use of exponential RSF, it is suggested in this work to select $n + 1$ sample points for first and second iterations only. Then, for the rest of iterations (until convergence), the latest estimated design point is replaced with one of the experimental points used in latest iteration. Here, this approach which considers the Euclidean distance between experiments, is called experiment updating technique and will be described in Section 3.3. These two improvements increase the efficiency of classical RSM. Furthermore, the accuracy of the RSM is improved, herein, by choosing the locations of sample points in a judicious way. This improvement, described in Section 3.1, is based on the vector length and the angle between two vectors. In some of the approaches proposed in literature for selection of experiment locations, it may lead, in some cases, to an ill-conditioned system of equations; this latter is avoided in the procedure proposed in Section 3.1. Several examples are finally solved in Section 4 to demonstrate the high efficiency and accuracy of the proposed method.

2. Classical RSM for reliability analysis

The reliability of a structure can be defined by the reliability index, i.e. the shortest distance between origin and limit state surface in standard Gaussian space. However, it may better be defined based on the probability of failure of the structure P_f, which is defined as

$$P_f = \int_{G(\boldsymbol{X})<0} f_X(\boldsymbol{X}) d\boldsymbol{X} \tag{1}$$

where $f_X(\boldsymbol{X})$ is the joint probability density function of the vector of basic random variables $\boldsymbol{X} = [x_1, x_2, \ldots x_n]^T$. $G(\boldsymbol{X})$ is the limit state function. The failure probability is integrated over the failure region defined as $G(\boldsymbol{X}) < 0$.

In classical RSM, in order to reduce the computational cost induced by evaluations of actual LSF, $G(\boldsymbol{X})$ is approximated by a RSF as $\tilde{G}(\boldsymbol{X})$. In conventional RSM, a quadratic polynomial function without cross terms [6] is chosen as RSF, which is expressed for the case of n random variables $\boldsymbol{X}$ as follows

$$\tilde{G}(\boldsymbol{X}) = a + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} c_i x_i^2 \tag{2}$$

wherein a, b_i and c_i are unknown coefficients to be determined using at least $2n + 1$ experimental points.

The experimental points are chosen to be the origin of standard space (corresponding to the mean values $\bar{\boldsymbol{X}}$) as the center point, and, $2n$ points on the n coordinate axes with a distance of f from origin. By using these $2n + 1$ points and solving $2n + 1$ linear equations, the unknown coefficients are obtained. Then, FORM is applied to find the MPP for the constructed RSF. Since, this point may not be true MPP of actual LSF, an iterative procedure is followed to find actual design point. For this aim, the RSF should be improved by updating the experimental points. Thus, the center point of experiments should be moved toward the estimated design point ($\boldsymbol{X}_D$). The center point of next iteration should lie on the actual LSF. To this end, an iterative procedure is followed: