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Improvement of equivalent component approach for reliability analyses of series systems

C. Gong, W. Zhou *

Department of Civil & Environmental Engineering, The University of Western Ontario, London N6A 5B9, Canada

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ABSTRACT

The present study proposes an improved equivalent component approach to evaluate the system reliability of series systems. An analytical expression is derived to evaluate the unit normal vector, in the context of the first-order reliability method, associated with the equivalent component. This improves the computational efficiency for determining the correlation coefficients between the system and equivalent components in the equivalent component approach. An adaptive combining process is also proposed, whereby the two components with the highest correlation coefficient are combined at each combining step. The accuracy and efficiency of the improved equivalent component approach are demonstrated for series systems with equally and unequally correlated components. Finally, the improved equivalent component approach is illustrated through the system reliability analysis of a pressurized steel pipeline segment containing ten active corrosion defects.

1. Introduction

Many engineering structures are characterized as series systems, e.g. pressurized pipelines containing multiple corrosion defects [1,2], bridge girders with different failure modes [3] and levee systems for flood defense [4]. Failure of any component of a series system leads to failure of the system. The first-order reliability method (FORM) [5–9] can be employed to evaluate the system reliability of series systems. Consider a series system consisting of m components. The application of the FORM to the jth ($j = 1, 2, \ldots, m$) component results in a linearized safety margin (i.e. a hyperplane to approximate the limit state surface) in the standard normal space and the corresponding reliability index β_j. The failure probability of the system, P_{fs}, is evaluated as $P_{fs} = 1 - \Phi_m(\boldsymbol{\beta}, \boldsymbol{R})$, where $\boldsymbol{\beta} = [\beta_1, \beta_2, \ldots, \beta_m]^T$; T denotes transposition; $\boldsymbol{R}$ is the $m \times m$ matrix of the correlation coefficients among the linearized safety margins for different components, and $\Phi_m(\bullet, \bullet)$ is the m-variate standard normal cumulative distribution function. The elements of $\boldsymbol{R}$, r_{jk} ($j, k = 1, 2, \ldots, m$), are computed as the inner product of the unit normal vectors associated with the jth and kth components obtained from the FORM [7,8].

Two key aspects of the FORM-based evaluation of P_{fs} are the computation of $\boldsymbol{R}$ and m-dimensional normal integral $\Phi_m(\boldsymbol{\beta}, \boldsymbol{R})$. A direct way to compute $\boldsymbol{R}$ is to carry out the FORM for each component by including all the random variables involved in the system. This ensures that the unit normal vectors for all components have the same dimension; the evaluation of r_{jk} then follows straightforwardly. Approaches for evaluating $\Phi_m(\boldsymbol{\beta}, \boldsymbol{R})$ are well reported in the literature [4,11–15]. Among them, the equivalent component approach [4,11,13,15] is the focus of the present study because of its ability to deal with systems with a large number of components. The basic idea of the equivalent component approach is to combine two components of the system into an equivalent component, which is then combined with a third component of the system. This process continues until a single equivalent component replaces all the components in the system.

Several variations of the equivalent component approach reported in the literature differ primarily in the way to evaluate the correlation coefficients between the equivalent component and remaining system components. In the approaches proposed by Gollwitzer and Rackwitz [11] as well as Estes and Frangopol [13], a linearized safety margin for the equivalent component is constructed in the standard normal space. The corresponding unit normal vector is then estimated from the finite difference method and used to evaluate the correlation coefficients between the equivalent component and remaining system components. The equivalent planes method (EPM) reported by Roscoe et al. [4] assumes that the same set of physical parameters are involved in different components. This assumption allows efficient evaluation of the unit normal vector of the linearized safety margin for the equivalent component through the finite difference method, but

* Corresponding author.
E-mail address: wzhou@eng.uwo.ca (W. Zhou).

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restricts the general applicability of the equivalent planes method. In the sequential compounding method (SCM) proposed by Kang and Song [15], the correlation coefficient between the equivalent component and a system component is evaluated by solving a nonlinear equation resulting from approximate decomposition of the bi- and tri-variate normal distributions using conditional probabilities.

In the present study, an analytical expression to evaluate the unit normal vector associated with the equivalent component is derived using the chain rule. The expression facilitates the evaluation of the correlation coefficients between the equivalent component and remaining system components. Moreover, an adaptive combining process for generating the equivalent component is proposed and shown to markedly improve the accuracy of the equivalent component approach for series systems with unequally correlated components. The remainder of the paper is organized as follows. The basics of the FORM in the context of the system reliability analysis are briefly described in Section 2. The improvement of the equivalent component approach is described in Section 3, and the illustration and validation of the proposed improvements in terms of numerical examples are presented in Section 4 followed by conclusions in Section 5.

2. FORM in the context of system reliability analysis

The basic concept of the FORM is well explained in the literature [5–9]. Consider the m-component series system described in the Introduction section. Let $g_j(\boldsymbol{x}_j)$ ($j = 1, 2, \ldots, m$) denote the limit state function associated with the jth component, where $\boldsymbol{x}_j$ is the value of $\boldsymbol{X}_j$ representing a vector of n_j random variables that need to be considered for $g_j(\boldsymbol{x}_j)$. Let $\boldsymbol{X}$ denote the union of all $\boldsymbol{X}_j$, representing a vector of n ($n \geq n_j$) random variables that need to be considered for the system. The value of $\boldsymbol{X}$ is denoted by $\boldsymbol{x}$. To consider the correlations among safety margins at different components in the system reliability evaluation, the reliability index β_j for the jth component can be evaluated by solving the following constrained optimization problem [7,8]:

$$\beta_j = \min_{g_{j,U}(\boldsymbol{u})=0} \sqrt{\boldsymbol{u}^{\mathrm{T}}\boldsymbol{u}} \tag{1}$$

where $\boldsymbol{u}$ denotes the value of the n-dimensional vector of independent standard normal variates $\boldsymbol{U}$ that is transformed from $\boldsymbol{X}$, and $g_{j,U}(\boldsymbol{u}) = g_j(\boldsymbol{x}(\boldsymbol{u}))$ is the limit state function in terms of $\boldsymbol{u}$. The techniques for transforming $\boldsymbol{X}$ to $\boldsymbol{U}$ have been reported in the literature [7–9]. For example, the Nataf transformation [10] can be used to first transform $\boldsymbol{X}$ to an n-dimensional vector, $\boldsymbol{Z}$, in the correlated standard normal space, and the transformation from $\boldsymbol{Z}$ to $\boldsymbol{U}$ is then achieved through $\boldsymbol{U} = \boldsymbol{L}^{-1}\boldsymbol{Z}$, where $\boldsymbol{L}$ is the lower-triangular matrix obtained from the Cholesky decomposition of the correlation matrix of $\boldsymbol{Z}$ [7]. The solution of $\boldsymbol{u}$ corresponding to Eq. (1) is the design point denoted as $\boldsymbol{u}^*$, and the corresponding unit normal vector, denoted by $\boldsymbol{\alpha}$, can be computed as $\boldsymbol{u}^*/\beta_j$. The unit normal vector is also known as the sensitivity factor as the elements of $\boldsymbol{\alpha}$ reflect the relative importance of the corresponding components of $\boldsymbol{u}$. The unit normal vector is then used to evaluate the correlation coefficients between the safety margins associated with different components.

The constrained optimization given by Eq. (1) is solved in the n-dimensional space for each β_j ($j = 1, 2, \ldots, m$). This can be computationally inefficient especially if n is large. A more efficient approach for evaluating β_j is recently proposed by Zhou et al. [16], who showed that β_j can be evaluated by only including $\boldsymbol{X}_j$ in the FORM analysis. The n_j-dimensional design point obtained from such an analysis is then mapped to the corresponding n-dimensional design point, which is subsequently used to evaluate the correlation coefficients between different components.

3. Improvement of equivalent component approach

3.1. Unit normal vector for equivalent component

Let $C_1, C_2, \ldots, C_m$ denote, respectively, the m components of the series system described in Section 2. The application of the equivalent component approach to the system is illustrated in Fig. 1, where $C^e_\bullet$ denotes an equivalent component. The first combining step results in the equivalent component C^e_{12}. Given the reliability indices β_1 and β_2 for C_1 and C_2, respectively, as well as the correlation coefficient between linearized safety margins at C_1 and C_2, r_{12}, the failure probability of C^e_{12}, P_{f12}, equals $1 - \Phi_2(\beta_1, \beta_2, r_{12})$ and is represented by an equivalent reliability index $\beta^e_{12} = -\Phi^{-1}(P_{f12})$, where $\Phi^{-1}(\bullet)$ is the inverse of the normal cumulative distribution function. To continue the combining process and generate the equivalent component C^e_{123}, the correlation coefficient between C^e_{12} and C_3, $r_{12,3}$, needs to be computed. This can be achieved by developing an equivalent linearized safety margin, $g^e_{12}(\boldsymbol{u})$, in the standard normal space for C^e_{12} [4,11,13]:

$$g^e_{12}(\boldsymbol{u}) = \beta^e_{12} - (\boldsymbol{\alpha}^e_{12})^{\mathrm{T}}\boldsymbol{u} \tag{2}$$

where $\boldsymbol{\alpha}^e_{12}$ is the n-dimensional unit normal vector associated with $g^e_{12}(\boldsymbol{u})$ and can be obtained as follows based on the sensitivity interpretation of the unit normal vector:

$$\boldsymbol{\alpha}^e_{12} = \frac{\partial \beta^e_{12}}{\partial \boldsymbol{u}} \Big/ \left\| \frac{\partial \beta^e_{12}}{\partial \boldsymbol{u}} \right\| \tag{3}$$

with $\|\bullet\|$ denoting the norm of a vector. The value of $r_{12,3}$ then equals $(\boldsymbol{\alpha}^e_{12})^{\mathrm{T}}\boldsymbol{\alpha}_3$. The finite difference method is generally used to evaluate $\boldsymbol{\alpha}^e_{12}$ in the literature [4,11,13]; however, this method is time consuming for systems involving a large number of random variables and may not be numerically robust.

In this study, an analytical expression utilizing the chain rule is developed to evaluate $\boldsymbol{\alpha}^e_{12}$. To this end, the ith ($i = 1, 2, \ldots, n$) element of $\frac{\partial \beta^e_{12}}{\partial \boldsymbol{u}}$, $\frac{\partial \beta^e_{12}}{\partial u_i}$, is given by

$$\frac{\partial \beta^e_{12}}{\partial u_i} = \frac{\partial \beta^e_{12}}{\partial P_{f12}} \frac{\partial P_{f12}}{\partial \beta_1} \frac{\partial \beta_1}{\partial u_i} + \frac{\partial \beta^e_{12}}{\partial P_{f12}} \frac{\partial P_{f12}}{\partial \beta_2} \frac{\partial \beta_2}{\partial u_i} \tag{4}$$

Note that

$$\frac{\partial \beta^e_{12}}{\partial P_{f12}} = -\frac{1}{\varphi(-\beta^e_{12})}, \tag{5}$$

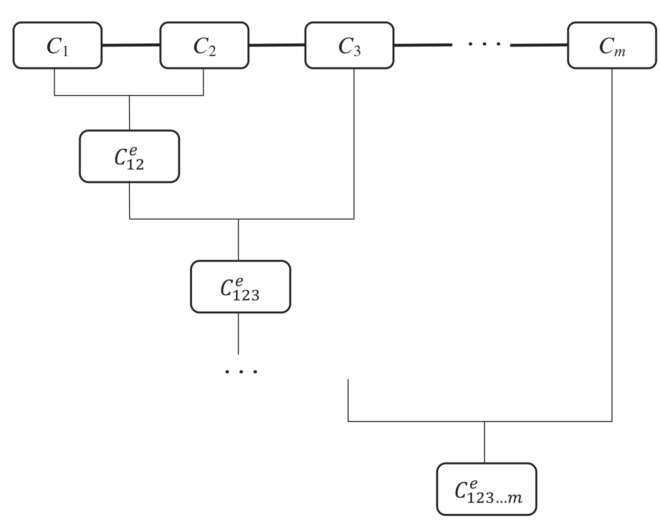

Fig. 1. Illustration of the equivalent component approach for a series system with m components.