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Structural reliability analysis based on probability and probability box hybrid model

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ABSTRACT

In many structural reliability analysis problems, probability approach is often used to quantify the uncertainty, while it needs a great amount of information to construct precise distributions of the uncertain parameters. However, in many practical engineering applications, distributions of some uncertain variables may not be precisely known due to lack of sufficient sample data. Hence, a complex hybrid reliability problem will be caused when the random and non-precise probability variables both exist in a same structure. In this paper, a new hybrid reliability analysis method is developed based on probability and probability box (p-box) models. Random distributions are used to deal with the uncertain parameters with sufficient information, while the probability box models are employed to deal with the non-precise probability variables. Due to the existence of the p-box parameters, a limit-state band will be resulted and the corresponding reliability index will belong to an interval instead of a fixed value. According to the interval analysis, the hybrid reliability model based on random and probability box variables is constructed and the complex nesting optimization problem will be involved in this hybrid reliability analysis. In order to obtain the minimal and maximal reliability index, the corresponding solution strategy is developed, in which the intergeneration projection genetic algorithm (IP-GA) with fine global convergence performance is employed as inner and outer optimization solver. Four numerical examples are investigated to demonstrate the effectiveness of the present method.

1. Introduction

In traditional structural reliability analysis, the probability model and the convex model are often used to describe the uncertain parameters. By using the probability approach, the failure probability of the engineering structure would be precisely known [1–6]. However, the accurate failure probability depends on a great amount of information on the uncertainty which is required to construct precise random distributions. Unfortunately, the distributions of some parameters may not be precisely known due to limited information in practical applications. By using the convex model approach, the lower and upper bounds of an uncertain parameter are only needed, not necessarily knowing its precise probability distribution compared with the probability approach. It seems that the structure reliability analysis can be made much more convenient and economical for practical engineering problems [7–13]. However, the convex model places excessive emphasis on the worst case of a structure, which would induce an overconservative description of the system variability and bring about some ultra-conservative designs.

As the literature survey reveals, probability model and convex model have their own merits and deficiencies. Thus, very naturally we wish to develop an effective reliability analysis model to remedy the deficiencies of the above traditional models. Therefore, the probability box model, or "p-box" for short, is introduced by Ferson et al. [14], which has been considered as beneficial supplements to the probability model and convex model. Probability box (p-box) is a rigorous and practical way to represent epistemic sources of uncertainty where the available knowledge is insufficient to construct the required probability distributions [15]. P-box has a much more flexible framework to quantify reliability analysis from the perspective of its theory, which is produced by combining probability theory and interval arithmetic [16–19]. Compared with the probability model and convex model, probability box model neither needs many samples, nor wastes useful

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information on the uncertainty. A series of prominent work in this filed has been carried out and reported. Destercke et al. [20] studied the relation between probability intervals and generalized probability boxes (p-boxes) and discussed transformation methods into probability intervals from p-boxes. Christopher [21] used probability boxes for cost uncertainty analysis as they can be easily derived from a small amount of samples. Sun et al. [22] presented a probability box methodology which was applied to a sewer flood evaluation problem. The methodology under probability box framework highlights the importance of separation of aleatory and epistemic uncertainties and preservation of the uncertainty nature. Luis et al. [23] presented a reliability analysis framework applicable to systems subject to polynomial requirement functions and probability box uncertainties. Other techniques to propagate probability box uncertainties in structural systems can also be found in Refs. [24–26]. In fact, the use of probability boxes in reliability analysis could offer many significant advantages over the traditional probability model and convex model. P-boxes can be regarded as a unified mathematical model to describe uncertainties with imprecise distributions or imprecise dependencies. Incorporating p-boxes into reliability analysis makes a wide range of practical problems with imprecise uncertainties solvable, which could be able to significantly relax the pre-assumption of using traditional probability approaches.

It is noteworthy that most of the above mentioned works focus on the single-type uncertain model. Actually, in many engineering problems, some of the uncertainties can be described with certain probability distributions and other uncertainties need to be treated as the non-probability convex model or probability box model due to their inherent natures or lack of sufficient sample data. Hence, hybrid model with both aleatory and epistemic variables is more common compared to the single-type model. Once mixed model exists, the traditional probability-based reliability analysis methods are not always available for many practical applications. Therefore, how to perform the hybrid reliability analysis for structures has been attracting more and more attention. In this field, many effective hybrid reliability analysis methods have been well developed based on that probability theory is integrated with the non probability convex theory [12,27,28], the evidence theory [29,30] and fuzzy sets theory [31,32]. Compared with above these works, however, relevant research on the hybrid reliability analysis based on probability and probability box models is relatively few [33–35]. Therefore, to utilize p-boxes in practical applications, the effective reliability analysis method based on probability and probability box hybrid model should be developed.

This paper aims to develop a new reliability analysis method based on probability and probability box hybrid model, which includes the construction of the hybrid reliability model and its effective solution algorithm. The remainder of this paper is organized as follows. The fundamentals of probability box theory and the construction of the probability and probability box hybrid model are introduced in Section 2. The solution strategy for this hybrid reliability model is proposed in Section 3. Four numerical examples are investigated to demonstrate the effectiveness of the present method in Section 4, respectively. Finally, some conclusions are summarized in Section 5.

2. Statement of the problem

2.1. Probability box model

A probability box (p-box) is an imprecise probability that is embraced by a lower and an upper bound on cumulative distribution function (CDF). P-box may be constructed in terms of any available information about an uncertain quantity, or may be generated from distributions of a specified shape but uncertain distribution parameters. In general, there are two main classes of models to describe the probability box models, which are named "distributional p-box model" and "Dempster-Shafer structure model".

In the distributional p-box model, the bounds on cumulative distribution function (CDF) of a probability box can be formulated by mathematical expressions which are known to have the particular shape. This p-box provides interval-like bounds on the cumulative distribution function (CDF) describing a probability distribution [14], which is represented by an upper (left) and a lower (right) cumulative distribution function $\bar{F}$ and $\underline{F}$, as shown in Fig. 1, where $\bar{F}$ and $\underline{F}$ are non-decreasing functions from the real line $\Re$ into [0,1]. For the cumulative distribution function $F_Y(Y) = p$, the corresponding p-box variable Y can be denoted by the inverse functions of $\bar{F}_Y^{-1}$ and $\underline{F}_Y^{-1}$:

$$\begin{aligned} \bar{F}_Y^{-1}(p) &= \{Y|\bar{F}_Y(Y) - p\}\forall p \in [0,1] \\ \underline{F}_Y^{-1}(p) &= \{Y|\underline{F}_Y(Y) - p\}\forall p \in [0,1] \end{aligned} \quad (1)$$

According to Eq. (1), for any cumulative probability $p \in [0,1]$ in a probability box, there is a corresponding interval $[\bar{F}_Y^{-1}(p), \underline{F}_Y^{-1}(p)]$ that maps from p.

Apparently, the above distributional p-box model could be described by the straightforward mathematical expressions of the cumulative distribution function (CDF). In practical engineering problems, however, the formula of the non-decreasing CDF $F_Y(Y)$ of the probability box model may not be precisely known due to limited information. To deal with the problems, Dempster-Shafer structure based on evidence theory [36,37] is used to describe the p-box, in which the cumulative distribution function edges $\bar{F}$ and $\underline{F}$ can be accomplished through a small quantity of information on the uncertainty such as mean, minimal and maximal values. Considering the Dempster-Shafer structure model as illustrated in Fig. 2, the lower (right) and upper (left) bounds of the p-box are shown as thick, black step functions from zero to one along the horizontal axis.

To find a Demspster-Shafer structure corresponding to this p-box, the discretization rectangles could be used to depict this model. The location of a rectangle along the horizontal axis defines a focal element of the Demspster-Shafer structure. The height m_i of each rectangle is the mass associated with that interval. Hence, the Dempster-Shafer structure also can be defined as a collection of pairs consisting of an interval and a mass $\{([a_1,b_1],m_1),([a_2,b_2],m_2),\ldots,([a_n,b_n],m_n)\}$, where $a_i \leqslant b_i$ for all i, $\sum m_i = 1$, and $b_i \neq b_j$ whenever $a_i = a_j$. Actually, when the p-box

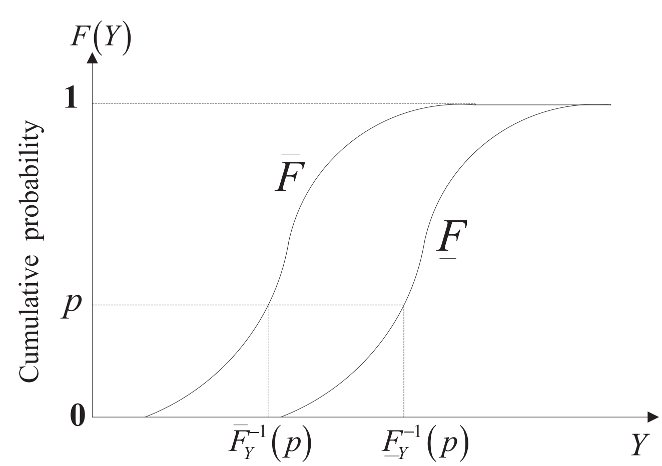

Fig. 1. A probability box consisting of a lower(right) and upper(left) bound.