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Stochastic renewal process models for estimation of damage cost over the life-cycle of a structure

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ABSTRACT

In the life-cycle cost analysis of a structure, the total cost of damage caused by external hazards like earthquakes, wind storms and flood is an important but highly uncertain component. In the literature, the expected damage cost is typically analyzed under the assumption of either the homogeneous Poisson process or the renewal process in an infinite time horizon (i.e., asymptotic solution). The paper reformulates the damage cost estimation problem as a compound renewal process and derives general solutions for the mean and variance of total cost, with and without discounting, over the life cycle of the structure. The paper highlights a fundamental property of the renewal process, referred to as renewal decomposition, which is a key to solving a wide range of life cycle analysis problems. The proposed formulation generalizes the results given in the literature, and it can be used to optimize the design and life cycle performance of structures.

1. Introduction

1.1. Background

The life-cycle cost analysis involves many elements, such as cost of construction, operation, maintenance, decommissioning, and many other activities, over a specified time horizon or service life of the structure. In the reliability-based optimization, Rosenblueth and Mendoza [1] pointed out the three most important components of the life cycle cost, namely, initial construction cost, benefits derived from the system and losses due to failures. The term *damage cost* is used in this paper to denoted the total losses due to failures that incur due to loss of services, damage to contents and cost of repairing and restoring the damaged structure.

In the life cycle analysis, one of the most uncertain elements is the damage cost that might result due to exposure to external hazards, such as earthquakes, wind storms and floods. Uncertainty in the estimation of damage cost arises from intrinsic uncertainties associated with the occurrence frequency and intensity of a given type of hazard, as well as the structural response to the hazard.

In recent times, research interests in the life cycle analysis has peaked, as it has become a focus of the performance-based design as well as optimization of decisions related to maintenance planning and retrofitting of structures.

In structural engineering, the homogeneous Poisson process (HPP) model for occurrences of a hazard has been traditionally used to estimate the expected life cycle damage cost, such as in the seismic risk analysis [2]. Although the HPP model greatly simplifies the analytical formulation, this model is not likely to represent the stochastic nature of a wide ranging hazards and threats. Therefore, the expected cost analysis performed under the HPP assumption cannot be considered a generic analysis of the problem.

The main aim of this paper is to provide a clear and comprehensive exposition of key ideas of the theory of stochastic renewal processes in a way to generalize the life-cycle analysis of the damage cost. In particular, derivations of the expected value and the variance of the cost, with and without discounting, are presented in a coherent manner. Explicit analytical results are derived for the HPP and Erlang processes, which are especial cases of general results presented in the paper. A practical example of seismic retrofitting is presented. An ulterior motive of this study is to help new generation of engineers understand the key concepts of stochastic process models for life-cycle cost analysis.

Since the paper is primarily concerned with damage cost resulting from external hazards, the effect of internal degradation (e.g., corrosion and fatigue) on the life cycle cost is not considered here.

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The inspection and maintenance costs to prevent failures resulting from internal degradation are also ignored. The topic of life cycle cost analysis considering a stochastic degradation process and a condition-based maintenance policy are already presented in separate studies by Cheng and Pandey [3] and Pandey et al. [4].

1.2. Literature

With the advent of probabilistic models for risk analysis in 1970s, there was a great deal of interest in using the total risk as a basis for optimizing the structural design codes. Whitman and Cornell [5] presented a comprehensive approach to evaluate the total seismic risk associated with a design that is expected to face multiple seismic events during its service life. The next comprehensive study on this topic was presented by Rosenblueth [6], who introduced the stochastic renewal process for estimating the expected present value of losses caused by infrequent hazards, such as earthquakes, strong winds and tsunamis. In this study, the expected discounted cost of structural failures and repairs was derived using the method of the Laplace transform. The problem of optimum design of structures under dead, live and seismic loads was considered in Rosenblueth [7]. The optimization involved minimization of expected discounted value of costs and losses over the life cycle of the structure. In earthquake engineering, calculation of lifetime seismic damage cost continued to be an active area of research [2], though the stochastic analysis is almost exclusively based on the homogeneous Poisson process (HPP) model. Porter et al. [8] presented computation of the variance of discounted seismic risk under the assumption of HPP model, perhaps the first time in the seismic literature. The derivation was based on the order statistics property of the Poisson process, which cannot be extended to a renewal process model.

Takahashi et al. [9] pointed out that the occurrence of large magnitude earthquakes, referred to as the ‘characteristic earthquake’ depends on the previous history of earthquake activity at the source. Therefore, a non-Poisson, non-stationary stochastic model must be used to describe their occurrences, whereas HPP model is more suitable for smaller earthquakes occurring more or less randomly. They adopted a renewal process model based on the Brownian Passage Time distribution and approximately evaluated the expected discounted cost of seismic damage. A detailed evaluation of structural damage and cost given a seismic event has also been an active area of research [10,11].

The interest in the renewal process model for life cycle cost optimization was rekindled by Rackwitz [12], in which Rosenblueth’s model was extended to combine it with the Life Quality Index framework proposed by Pandey et al. [13]. In a series of papers, Rackwitz and his co-workers applied the renewal process model to a more general class of problems in which the effect of degradation and maintenance was also included in life cycle cost analysis [14–16]. Most of this work was concerned with the evaluation of expected discounted cost and losses. Goda and Hong [17] applied the Monte Carlo simulation method to evaluate the mean, standard deviation and probability distribution of the seismic life cycle cost. An application of the utility theory to life cycle analysis was presented by Cha and Ellingwood [18].

1.3. Limitations of existing literature

Although there is a fairly substantial body of the literature on stochastic modeling of life cycle cost analysis, the following limitations in the analytical formulation are noted:

- In the stochastic life cycle analysis, the homogeneous Poisson process model is omnipresent [8,12,2]. The HPP model leads to considerable analytical simplifications and avoids dealing with intricacies of the theory of the renewal process.

  The analysis is mostly limited to the expected cost and expected discounted cost. The computation of the variance is largely no-existent, with an exception of Porter et al. [8], who derived variance of the cost.
- Although the stochastic renewal process models were employed hitherto, their success has been mostly limited to the computation of expected cost in an asymptotic sense. In fact, a clear formulation for the expected discounted cost in a finite time horizon is not available.

  The asymptotic analysis is based on the elementary renewal theorem which says that the cost rate asymptotically converges to a ratio of the expected cost in a single renewal cycle to the expected cycle length. This asymptotically solution is so simple to use that it completely bypasses a formal stochastic formulation of the problem. For this reason, the literature is replete with the use of the asymptotic solution, even in cases where it is not consistent with a short and finite time planning horizon, required for financial planning and capital budgeting [3].
- The evaluation of variance of the life cycle cost and its discounted value in a stochastic renewal model has not been discussed at all in the life cycle analysis literature.

  A main reason for lack of generalities in renewal process based models is the method of the Laplace Transform that was used by most researchers to solve the problem [12,6]. Although this method allows to write a compact expression for the Laplace transform of the expected costs, its inverse in not easy to find for a general distribution of the inter-occurrence time. Therefore, this approach is mostly limited to a few special cases like the exponential distribution (i.e., HPP model) and the Erlang distribution.

1.4. Objectives and organization

The central objective of this paper is to present a clear and comprehensive formulation to compute expected value and variance of the damage cost, with and without discounting, that may incur over the life cycle of a structure due to exposure to external hazards like earthquake, wind, snow and flood. To achieve this objective, a general formulation based on the theory of stochastic renewal process is presented, which overcomes the limitations of the existing literature as stated in Section 1.3. The mean and variance of discounted cost can now be computed in a finite time horizon for a general renewal process.

The information about the mean and variance of life cycle cost can be used to improve decision making regarding the design alternatives and options of retrofitting of a structure within a “mean–variance” based utility framework. For example, a utility function given as the sum of mean and some multiple of standard deviation of cost can be maximized as a part of the decision making process.

In this paper, analytical results are also derived for a especial case of the Erlang renewal process. An interesting finding of the paper is that there is large variability associated with the estimate of the damage cost, as marked by a large coefficient of variation (COV ≈ 1). It means that an exclusive reliance on the expected cost in optimization would not yield desired result in practice due to potentially large variability in the actual outcome.

The paper is organized as follows. Section 2 presents the basic terminology and concepts of the stochastic renewal process model. The renewal decomposition, a fundamental concept used extensively in this paper, is clearly described. The lack of understanding of this key concept led many researches to adopt the Laplace Transform approach. Section 3 derives the expected cost and variance of the damage cost, and this formulation is extended to discounted cost analysis in Section 4. Analytical results for HPP and the Erlang renewal process are derived in Section 5. A practical example