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Interaction of axial force and bending moment by using Bouc-Wen hysteresis and stochastic linearization

Felice Colangelo *

University of L'Aquila, DICEAA Department, Via Giovanni Gronchi 18, 67100 L'Aquila (AQ), Italy

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ABSTRACT

Among the probabilistic methods of analysis of multi-degree-of-freedom nonlinear structural systems, stochastic equivalent linearization is a common alternative to Monte Carlo simulation. Using the Bouc-Wen model in particular, may yield the equivalent system in analytical closed form, which is fundamental to efficient computation. Within this context, the Bouc-Wen model is here extended in a simple fashion to introduce the interaction of the resisting axial force and bending moment, as is typical of short columns. The member is idealized as a massless linearly elastic beam element provided with terminal rotational springs whose behavior follows the extended Bouc-Wen model. Consistent with a parabolic interaction diagram, the probabilistic moments of the response are formulated by the common stationary Gaussian nonzero-mean linearization method with random earthquake motion, deterministic gravity load and asymmetric hysteresis. The interaction model is validated by comparing the response of two portal frames, whose different sensitivity to interaction is captured. In addition, Monte Carlo simulation is carried out using a piecewise linear interaction model. The effect of parabolic interaction on the displacement and hysteretic rotation from linearization does not match quantitatively the piecewise-linear interaction in the simulation. Nevertheless, qualitative agreement is satisfactory. Full agreement appears between other response quantities. The proposed interaction model proves to be suitable at least for comparative probabilistic seismic analyses of framed structures with possible yielding of columns.

1. Introduction

The seismic analysis of buildings involves randomness of the ground motion as well as unpredictability of the hysteretic structural behavior, not to mention the uncertainty from fuzziness and partial knowledge. A probabilistic approach is still too difficult and computationally demanding for engineering practice, but necessary to research on underlying issues. Many methods of stochastic structural analysis are now available. Concerning the randomness of excitation, such methods may be classified as oriented to the numerical characteristics or to the probability density function (PDF); about the structural uncertainty, one can distinguish the perturbation approach, the orthogonal expansion, and Monte Carlo (MC) simulation [1]. To the former class belongs the stochastic equivalent linearization (SL) method. In brief, any nonlinear relationship in the structure is replaced by a linear one, optimum in some statistical sense [2,3]. Clearly, a certain approximation is inevitable; inherent error depending on the degree of nonlinearity as well as other factors is a drawback [4–7]. Nevertheless, the SL method is deemed to be the only feasible alternative to MC simulation where the computational burden is a concern, for instance to analyze the multi-degree-of-freedom (DOF) hysteretic systems of engineering practice [3,8–14]. In some sense, the SL method is opposite to MC simulation, with most of the other methods being placed in-between [7].

Bouc [15] and Wen [16] are authors of a well-known smooth differential model suitable for representing a variety of nonlinearity and hysteresis. Despite its great versatility, this behavior model may be stochastically equivalent to a linear system in analytical closed form, conditional on the sense of equivalence as well as advisable parameter values [17]. Clearly, knowing the parameters of the equivalent system as explicit functions of the probabilistic moments of the response is a key point for any efficient computation. This is why a number of extensions of the Bouc-Wen model appear in the literature with frequent application to SL analysis [3,18]. In brief, Baber and Wen [19] and Sues et al. [20] introduced cyclic degradation. Further study by Baber and Noori [21] and Foliente [22] led to include pinching. Recent elaboration of the degrading pinching model is by Bursi et al. [23], Sengupta and Li [24] and Kottari et al. [25]. Park et al. [26] covered biaxial bending. Wang and Wen [27] generalized such extension, finally improved

* Corresponding author.
E-mail address: felice.colangelo@univaq.it

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by Harvey and Gavin [28]. Considerable attention has been given to asymmetry. Proposals with various flexibility and complexity come from Baber and Noori [29], Colangelo et al. [30], Dobson et al. [31], Wang and Wen [27], Song and Der Kiureghian [32,33], Kwok et al. [34] and Zhu and Wang [35]. Shih and Sung [36] implemented isotropic hardening. Sireteanu et al. [37] and Love et al. [38] introduced evolution from softening to hardening behavior. Miah et al. [39] made the pre- to post-yield transition versatile. Finally, it is noteworthy that the Ozdemir model [40], used often as a basis to model superelasticity [41–43], may be seen as a special case of the Bouc-Wen model [44,45].

Baber and Wen [46,47] formulated a lumped-plasticity model for the seismic SL analysis of hysteretic framed structures. This model consists of linearly elastic beam elements provided with zero-length rotational springs at the ends. The moment and rotation of the springs follow the Bouc-Wen equation. This study focuses on extending such model in a simple fashion to make the flexural strength of the springs dependent on the axial force in the beam element. The interaction between the resisting axial force and bending moment (PM interaction) typical of a short column is introduced. Such a feature may be crucial, for instance under vertical ground motion [48,49]. In the field of SL, a piecewise linear interaction of the biaxial bending moments has long been formulated following a general multivariate approach, which in principle can be applied to the PM interaction as well [8]. However, to the knowledge of the writer any implementation into the Bouc-Wen model is missing. Herein the PM interaction is simplified as parabolic and incorporated into the Bouc-Wen model made asymmetric by Colangelo et al. [30]. The stochastically equivalent linear system is formulated in analytical closed form, preserving efficient computation. As a first step, the common Gaussian SL method is considered. The interaction model is validated on numerical basis. The results from the stationary SL analysis of two portal frames, differing in the importance of PM interaction, are compared with each other. Such results are also compared with MC simulation using a piecewise linear interaction model.

2. Proposed model

The proposal from this study consists of (i) the extension of the Bouc-Wen model, and (ii) the incorporation into the finite element model of a framed structure. This is detailed in the next sections, respectively.

2.1. Extended Bouc-Wen model

As usual, a hysteretic component is connected in parallel with a linearly elastic component. The behavior model in terms of total bending moment M and flexural rotation θ of a rotational spring can be written as

$$M = \alpha k_\theta \theta + (1-\alpha) k_z z \tag{1}$$

where k_θ = stiffness of the linear component; k_z = alike parameter of the hysteretic component; α = parameter to give weight to each component, related to the post-yield hardening ratio; z = auxiliary variable to formulate hysteresis. Herein any degradation is not considered as the stationary analysis is of concern; pinching is neglected as well. In practice, the framed structures where flexure dominates and detailing is good are expected not to show such features. The Bouc-Wen model extended to introduce PM interaction is

$$\dot{z} = \dot{\theta}\Big\{a y^n(P) - |z|^n\Big[\gamma + \beta \operatorname{sgn}(z\dot{\theta}) + \delta \operatorname{sgn}(z)\Big]\Big\} \tag{2}$$

where a, n, γ, β = parameters as in the original Bouc-Wen model. In a few words, a is related to the slope, n governs the smoothness of yielding, the sign of γ dictates softening or hardening behavior and β introduces the hysteresis by changing the stiffness between loading and unloading. Indeed, there is redundance and most features do not depend on a single parameter each; a proper interpretation requires normalization [50]. δ = additional parameter for the asymmetry according to Colangelo et al. [30]; $\operatorname{sgn}(\cdot)$ = signum function; $y(P)$ = function of axial force that introduces PM interaction. In fact, the positive and negative asymptotic values of the auxiliary variable, related to the resisting bending moments, are

$$z_y^{(+)},\ |z_y^{(-)}| = \left(\frac{a}{\gamma+\beta\pm\delta}\right)^{1/n} y(P) \tag{3}$$

Setting $y(P) \equiv 1$ gives the previous model without PM interaction, symmetric ($\delta = 0$) or asymmetric ($\delta \neq 0$; the sign of δ dictates which resistance is stronger). Instead, it is suggested to assume the following parabola

$$y(P) = 1 - \left(\frac{2P - P_{\max} - P_{\min}}{P_{\max} - P_{\min}}\right)^2 \qquad P_{\min} \leqslant P \leqslant P_{\max} \tag{4}$$

where $P_{\min}$, $P_{\max}$ = extreme resisting axial forces at pure tension and compression failure, respectively. Despite its simplicity, a parabolic function may be a sound approximation of the interaction diagram, as shown below. Moreover, a parabolic function is efficient for the SL analysis as in Section 3. Relevant expectations can be reduced to analytical closed form single expressions, whereas a piecewise linear function, for example, would require fragmentation of integrals. This choice is also consistent with using a smooth hysteretic model, whereas a piecewise one would lead to irreducible integrals [6]. It is noteworthy that $y(P)$ as in Eq. (4) is dimensionless and bounded by 0 and 1; the parameters of the former model, including δ for possible asymmetry, govern the dimensional flexural strength.

A limitation is that Eq. (4) requires negligible probability of the axial force being outside the interval [$P_{\min}$, $P_{\max}$]. This is not guaranteed with a Gaussian process, as in the most popular SL method used in Section 3. The axial force must be relatively small; failure controlled by extreme axial forces is not covered. Moreover, Eqs. (1)–(4) imply parabolic approximation of the PM interaction diagram. This is usual for steel members. In the case of reinforced concrete, the same approximation may be good (Fig. 1(a)) or poor, especially for asymmetric cross sections (Fig. 1(b)). In this respect, if the PDF of axial force is negligible outside some interval smaller than [$P_{\min}$, $P_{\max}$], the parabolic approximation may be fitted within that reduced interval (Fig. 1(c)). Finally, $y(P)$ into Eq. (2) affects not only the strength but, accidentally, also the stiffness. The exponent n should be great to minimize violation of the plasticity postulates, while the elasto-plastic behavior is approached more and more [51,52]. As the exponent n increases, the stiffness decreases, which is not very rational. Clearly, the proposed model is quite simple and conceived for engineering use.

2.2. Framed structure model

Planar framed structures are considered. The beam elements have arbitrary orientation and axial, flexural and shear linearly elastic behavior. The plastic deformation may occur in zero-length rotational springs at any end of each beam element, according to Eqs. (1)–(4). Since the seismic response, e.g. stress resultants and yielding itself, is random, every possible critical region should be provided with a hysteretic spring. Each spring implies that one rotation θ and one auxiliary variable z are introduced along with the nodal DOF's, which is relatively demanding. Indeed, there is a sound criterion for detecting those springs unlikely to yield, leading to remove relevant nonlinear equations [53]. Such a criterion is based on a frequency domain method that is not used in this study. Obviously, the initial stiffness of the spring should be so great that the rotation θ remains small unless actual yielding