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Imprecise probability analysis of steel structures subject to atmospheric corrosion

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ABSTRACT

Evaluating the behaviour of deteriorating steel structures is complicated by the inherent uncertainties in the corrosion process. Theoretically, these uncertainties can be modeled using a probabilistic approach. However, there are practical difficulties in identifying the probabilistic model for the deterioration process as the actual corrosion data are rather limited. Also, the dependencies between different random variables are often vaguely known and, thus, not included in the modeling. This paper proposes a probabilistic analysis framework for modeling the atmospheric corrosion of steel structures with incomplete information. The framework is based on the theory of imprecise probability and copula. Two examples are presented to illustrate the methodology. The role of epistemic uncertainties on structural reliability is investigated through the examples.

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1. Introduction

For the safety assessment of deteriorating steel structures, it is crucial to develop a reliable probabilistic model of deterioration to predict the temporal changes to structural resistance [1,2]. The deterioration of steel structures is a stochastic process with high uncertainties and variabilities. Recent works have treated the uncertainties using a pure probabilistic approach [3,4]. This approach requires that all statistical characteristics for each uncertainty can be determined reliably from sufficient observational data. In practice, however, available real-world data on structural corrosion are very limited, and the selection of probabilistic models (e.g., distribution type and/or distribution parameters) for uncertain variables is so generally based on limited information and/or subjective judgment.

It is thus advisable to consider the distribution itself as uncertain when the available data is limited. Statistical estimations provide us with distribution functions for the sampling uncertainty, which depends on the sample size. This uncertainty is reducible with an increasing amount of information/data. From this angle, it may be understood as epistemic uncertainty. Within a pure probabilistic framework, epistemic uncertainty can be handled with Bayesian approaches. Uncertain parameters of a probabilistic model can be described with prior distributions and updated by means of even limited data. They can then be modeled by Bayesian random variables and introduced formally, together with the remaining (aleatory) uncertainties, in the probabilistic analysis [5]. Judgmental information is needed to characterize the epistemic uncertainties. The characterization of the epistemic uncertainties can be substantiated by using the Bayesian updating rule when data become available. However, when the data is very limited, the result of the Bayesian approach remains as almost purely subjective.

Alternatively, an imprecisely known probability distribution can be modeled by a family of all candidate probability distributions which are compatible with available data. This is the idea of the theory of imprecise probabilities [6]. Dealing with a set of probability distributions is essentially different from a Bayesian approach. A practical way to represent the distribution family is to use a probability bounding approach by specifying the lower

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and upper bounds of the imprecise probability distribution. This corresponds to the use of an interval to represent an unknown but bounded number. Consequently, a unique failure probability cannot be determined. Instead, the failure probability is obtained as an interval whose width reflects the imprecision of the distribution model in the calculated reliability.

A popular uncertainty model using the probability bounding approach is the probability box (p-box for short) structure [7]. A p-box is closely related to other set-based uncertainty models such as random sets, fuzzy probabilities, Dempster-Shafer evidence theory and random intervals. In many cases, these uncertainty models can be converted into each other, and thus considered to be equivalent [7–10]. Therefore, the p-box approach presented in this paper is also applicable to other set-based uncertainty models. The approach of imprecise probability generally requires less subjective information than the Bayesian approach. It can be argued that, from a frequentist point of view, the epistemic uncertainties in the probability distribution can be more faithfully represented using a probability bounding approach [6,7,11].

Conventional probabilistic analysis often neglects the correlations and dependencies between random variables. This assumption is a common practice partly due to its mathematical convenience, but more likely due to the limited availability of data. It has been shown that the wrong assumption of dependence can lead to unreliable predictions for risk assessments [12]. Copula theory is a powerful tool for the dependence modeling of multivariate data. A copula is a joint cumulative distribution function (CDF) with uniform marginal. Copula theory has been used to model dependence in probability boxes. Ref. [12] proposed a dependence bounds convolution approach in which the uncertainties are modelled as Dempster-Shafer structures and the dependence is expressed as a given parametric copula. This method is useful for calculations of basic arithmetic operations with small numbers of variables. In [13], copula theory is combined with random sets for computing the lower and upper bounds of a failure probability.

This paper proposes a practical framework for uncertainty analysis using dependent p-boxes in which copulas describe the dependence. The Akaike Information Criterion is used to select the copula model that provides the best fit to the observational data. The confidence intervals of the copula parameter are estimated using the Bootstrap method. The dependent p-boxes are propagated through interval Monte Carlo (MC) simulation in order to assess structural reliability. The framework is applied to the time-dependent reliability analysis of steel structures subject to atmospheric correlations, and is demonstrated through two examples. The importance of epistemic uncertainty in the probabilistic modeling including dependencies is demonstrated on its influence on the reliability estimates.

2. Dependent probability boxes

2.1. Probability boxes with dependencies

Let $F_X(x)$ denote the cumulative distribution function (CDF) for a real-valued random variable X. A probability box is defined by a pair of CDFs, $\underline{F}_X(x)$ and $\overline{F}_X(x)$, which form the envelopes of the probability family

$$\mathcal{P} = \{P | \forall x \in \mathbb{R}, \underline{F}_X(x) \leqslant F_X(x) \leqslant \overline{F}_X(x)\}. \tag{1}$$

A p-box thus represents an $F_X()$ which is imprecisely known except that it is within the two bounding CDFs. It can be seen that $\underline{F}_X()$ and $\overline{F}_X()$ are the lower and upper probabilities of the event $X \leqslant x$. Detailed background can be found elsewhere [7]. There are various ways to define p-boxes such as utilizing Kologorox-Smirnow (K-S) confidence limits, Chebyshev's inequality, or by distributions with interval parameters, depending on the amount of available information [14].

The modeling of dependencies between probability boxes follows the concept of dependence between random variables. Both Pearson correlation and rank correlation have been adopted for p-boxes, but retaining their limitations known from probability theory. Thus, copula models have been suggested to describe dependence between p-boxes [15]. There are two main advantages of using copulas for this purpose. First, copulas can account for various types of dependencies. Second, the copula is flexible in selecting the appropriate dependence model independently from choosing the marginal distributions for each variable [16].

2.2. A brief introduction of copulas

A copula is a multivariate CDF for which the marginal distribution of each variable is uniform. According to Sklar's Theorem, a joint distribution can be expressed in terms of the marginal distribution functions and a copula which describes the dependence structure between the variables. Consider a d-dimensional random vector $\mathbf{X} = (X_1, X_2, \ldots, X_d)$ with margins $F_i(x), i = 1, \ldots, d$. There exists a copula C such that the joint CDF, denoted by $F_{\mathbf{X}}(x_1, \ldots, x_d)$, can be written as

$$F_{\mathbf{X}}(x_1, \ldots, x_d) = C(F_1(x_1), \ldots, F_d(x_d)). \tag{2}$$

There are two common classes of copulas; Gaussian and Archimedean. The Gaussian copula is used for the normal dependence structure. This structure can be estimated from its only parameter of a correlation matrix [17]. In a non-normal case, Archimedean copulas are often used to model the dependence structure in the data. The class of copula has a closed-form of representation,

$$C(u_1, u_2, \ldots, u_d, \theta) = \varphi^{-1}(\varphi(u_1), \varphi(u_2), \ldots, \varphi(u_d, \theta)), \tag{3}$$

in which $\varphi$is a generator with φ^{-1} completely monotonic on $[0, \infty) \times [0, \infty) \ldots \times [0, \infty)$ (d-dimensional copula). The copula parameter, θ, can be related to various dependence structures of Archimedean copulas. The most common Archimedean copulas include Clayton, Gumbel and Frank copulas which are summarised in Table 1. Details about copulas can be found elsewhere, e.g., [18].

2.3. Estimation of copula parameter

Different copulas represent different dependence structures on the data. Thus, we establish the copula model in two steps. Step 1 is devoted to estimate the parameters for a number of candidate copulas. The copulas considered in this paper (e.g., Clayton, Gumbel and Frank copulas) involve only one parameter, denoted by θ. The copula parameter θ can be estimated by the classical maximum likelihood estimation (MLE). The MLE yields a point estimate of θ.

In step 2 the best-fit copula model for the given observed data and (point-) estimated parameter is selected. This is realized based on the Akaike Information Criterion (AIC), which has particular

Table 1
Some common Archimedean copulas.

Copula	Form	Range of θ
Clayton	$C(u_1, u_2, \theta) = (u_1{}^{-\theta} + u_2^{-\theta} - 1)^{-1/\theta}$	$(0, \infty)$
Frank	$C(u_1, u_2, \theta) = -\theta^{-1} \log\left\{1 + \frac{(e^{-\theta u_1} - 1)(e^{-\theta u_2} - 1)}{e^{-\theta} - 1}\right\}$	$\mathbb{R}$
Gumbel	$C(u_1, u_2, \theta) = \exp\left(-\left((-\log(u_1))^{\theta} + (-\log(u_1))^{\theta}\right)^{1/\theta}\right)$	$[1, \infty)$

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