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Topology optimization for linear stationary stochastic dynamics: Applications to frame structures

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ABSTRACT

This work develops some foundations of topology optimization for the robust design of structural systems subjected to general stationary stochastic dynamic loads. Three methods are explored to evaluate the dynamic response – the time domain, frequency domain, and state space methods – and the associated design variable sensitivities are derived analytically. The resulting stochastic dynamic topology optimization problem is solved using the gradient-based optimizer Method of Moving Asymptotes (MMA). Sensitivities are computed using the adjoint method and the popular Solid Isotropic Material with Penalization (SIMP) is used to achieve clear existence of structural members. The approach is used to design the lateral load systems of structures that minimize the variance of the system response to stationary stochastic ground motion excitation. Numerical results are presented to illustrate the differences between topologies optimized for stochastic ground motion and topologies optimized for equivalent static loading.

1. Introduction

With applications from the design of material micro-structures to large scale mechanical systems, topology optimization is advancing rapidly as a form-finding methodology for load-carrying systems. As the vast majority of work to date considers static and deterministic loads, here we propose the foundations for applying topology optimization to systems subjected to random vibrations. The goal is to identify a design that, to the largest extent possible, mitigates the effects of stochastic dynamic excitation through an optimal structural configuration given a set of constraints. The methodology is explored herein through the optimization of building frame systems subjected to stationary stochastic ground motions.

There is a rich history of structural optimization of frames and trusses for static loads, dating back to the early 20th century when Michell [1] used Maxwell's Theorem to design uniformly stressed frames under single load cases. These so-called Michell structures have been studied extensively by a number of researchers [2–5] and have been used to guide structural design in practice [6]. More generally, topology optimization has been applied as a free-form methodology to design building frame structures under static loads. Mijar et al. [7], and later Liang et al. [8], used continuum topology optimization to establish "conceptual" designs for the lateral bracing of pre-defined frame structures. More recently, Stromberg et al. [9] applied pattern constraints to achieve repetition of a local lateral bracing topology along a building height.

Beyond deterministic static topology optimization, the concept of reliability-based topology optimization (RBTO) for static loadings have attracted a great deal of interest. In the context of RBTO, Kharmanda et al. [10] were among the first to consider reliability analysis in the objective function. This has subsequently been extended to the design of, for example, MEMS devices [11], trusses with geometric imperfections [12], and geometrically nonlinear structures using both probabilistic [13] and non-probabilistic methods [14]. Meanwhile, the more broadly-define topic of reliability-based design optimization has attracted much attention in the past 20 years. Although we are specifically interested in topology optimization-based approaches in this work, the reader may be interested in works such as those by Yuon and Choi [15] who consider a variety of probabilistically posed constraints, Kang et al. [16] who present a non-probabilistic approach, Maute et al. [17] who employ the spectral stochastic finite element method, Li and Au [18] who proposed a simulation based approach using subset simulations, and Cho and Lee [19] who propose the sequential optimization and reliability assessment method, to name just a few.

While the efforts to optimize structures for static conditions have been inspiring, the next generation of structural optimization must consider the true dynamic conditions of the loads placed on our structures. To this end, several researchers have considered structural optimization in a dynamic setting. This has included maximizing the natural frequency of structures in free vibration (e.g. [20]), as well as considering deterministic dynamic loads with

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structural response computed in the time or frequency domain. Lim et al. [21] performed dynamic response optimization using an active set recursive quadratic programming (RQP) algorithm. Chahande et al. [22] and Min et al. [23] minimize the dynamic compliance under dynamic loads using time history analysis. Wang et al. [24] investigated several formulations of 1st order and 2nd order differential equations for transient dynamic response optimization. Spence et al. [25] considered dynamic response optimization of structures under wind loads. The sensitivity analysis required to guide design updates under transient loads has been studied in several works ([26–29]), with particular discussion on the computational aspects available in [30]. Examples of structural optimization in the frequency domain include the work of Ma et al. [31] who used Optimality Criteria methods and, for topology optimization, the work of Yoon et al. [32] who used model reduction schemes.

The nature of the dynamic environment in which real structures operate are often highly uncertain. This has given rise to the vast study of stochastic dynamics in which a structure (deterministic or stochastic) is subjected to dynamic loading represented as a stochastic process. Such analysis is common, for example, for evaluating the structural response to seismic ground motion or wind pressure, determining response of machine components in a vibratory environment, or in the assessment of vehicle response (e.g. automobile suspension systems, aircraft frames, ship hulls, etc.). While some topology optimization researchers have begun considering uncertainties under static loads, including uncertainties in load magnitudes and directions (e.g., [33–36]) and uncertainties in structural geometry and material properties (e.g., [37–39,12]), only a few researchers have considered the optimization of structures subject to the random dynamic excitation that is common for many real structures, and nearly all of these works are limited to the case of white noise excitations. For example, Yang et al. [40] studied the optimal design of passive energy dissipation systems using performance metrics. Rong et al. [41] used the Evolutionary Structural Optimization (ESO) method to optimize continuum structures under white noise excitations in the frequency domain while Pagnacco et al. [42] similarly investigated ESO of structures subject to white noise with fatigue constraints – also in the frequency domain. Recently, Lin et al. [43] used topology optimization to design piezoelectric energy harvesting devices subjected to white noise random vibrations. Qiao et al. [44] considered both static loads and white noise stochastic loads for the layout optimization of multi-component structures. Rong et al. [45] also developed a sequential quadratic programming (SQP) method for topology optimization of structures under white noise excitation. Those that are not confined to white noise excitations, such as the works by Taflanidis and Scruggs [46] and Gidaris and Taflanidis [47] do not explicitly consider the structural topology itself in the optimization. Instead, these works aim to optimize the inter-story damping coefficients which, while important, does not afford the design flexibility of topology optimization. To the authors' knowledge, the only existing work that considers topology optimization for non-white noise stochastic dynamics is the work of Bobby et al. [48] who optimize the structural weight subject to certain performance metrics which requires reducing the structural system and performing topology optimization on an approximate sub-problem.

This optimization under non-white noise stochastic dynamic loading is the context of the current work. Specifically, we develop the mathematical framework for topology optimization of linear structures subject to general stationary Gaussian stochastic dynamic excitation. We take, as the objective of our optimization, the minimization of the response variance at a point on the structure and construct three alternative methods based on time domain, frequency domain, and state space methods for performing the sensitivity analysis to drive the design optimization. The analytical sensitivity analysis in the state space solution is enabled by an efficient algorithm for modal decomposition to calculation stochastic response developed by Igusa [49]. The resulting algorithm is then demonstrated on braced frame systems subject to stochastic base excitation and discussion of the methods and results are provided throughout.

2. Topology optimization formulation

The goal of topology optimization is to optimally distribute material within a design domain. In the case of structural building design, this typically means determining the location, size and connectivity of structural members within the building framing system. For frames (and trusses), the topology optimization problem is often formulated using the ground structure approach where the design domain is densely meshed with frame (or truss) elements and the optimization algorithm is used to determine which elements are to remain in the final topology. Although existence problems are discrete (a member exists or it does not exist), we relax the discrete condition to allow use of gradient-based optimization methods and drive the continuous design variables to binary solutions using the popular Solid Isotropic Material with Penalization (SIMP) [50]. Although typically used in continuum domains, including to design conceptual lateral bracing in buildings (e.g., [7]), the approach is illustrated by optimizing the bracing scheme in frame structures. Sensitivities are computed to determine local descent directions of the constrained objective function at every iteration, and these are then utilized by the gradient-based optimizer Method of Moving Asymptotes (MMA) [51,52] to compute design step decisions.

In this work, we focus on robust design and aim to minimize the variance of response quantity $Z(t)$ representing the stochastic response of a structure subject to stationary and Gaussian random vibration under the condition that the total material usage is limited. More formally, this is expressed as

$$\begin{aligned} \min \quad & f = \sigma_Z^2 \\ s.t. \quad & (\mathbf{K}(\boldsymbol{\rho}) - \omega_i^2 \mathbf{M})\boldsymbol{\phi}_i = 0, \quad \text{for } i = 1, \ldots, n \\ & \boldsymbol{\rho}^{\mathrm{T}}\mathbf{v} \leqslant V_{\max} \\ & 0 \leqslant \rho_e \leqslant 1 \quad \forall e \in \Omega \end{aligned} \tag{1}$$

where response quantity $Z(t)$ can be any response quantity that is a linear function of the modal coordinates. In our applications, it is a displacement. $\mathbf{K}$ is the structural stiffness matrix, $\mathbf{M}$ is the mass matrix, ω_i are the generalized eigenvalues, $\boldsymbol{\phi}_i$ are the generalized eigenvectors, $\boldsymbol{\rho}$ is the vector of design variables, and ρ_e is the magnitude of the design variable for element e. The second constraint is the material usage constraint with $\mathbf{v}$ being the vector of elemental volumes and $V_{\max}$ the maximum allowable volume of material. The last set of constraints over the design space Ω are design variable bounds where $\rho_e = 1$ indicates the element exists in the final topology and $\rho_e = 0$ indicates element removal.

As previously stated, the popular SIMP method is employed to achieve approximately binary design variables. In the SIMP method, the element stiffness matrix is defined as

$$\mathbf{K}_e(\boldsymbol{\rho}_e) = \left(\rho_e^{\eta} + \rho_{e,\min}\right)\mathbf{K}_{e0} \tag{2}$$

where the exponent η is used to ensure $\rho_e \to 0, 1$, $\rho_{e,\min}$ is a nominal minimum value (set to 1e-4) to guard against singularities in $\mathbf{K}$, and $\mathbf{K}_{e0}$ is the nominal element stiffness given its full attributes (i.e. $\rho_e = 1$). A continuation method is used on the SIMP exponent to help avoid local minima, as is commonly done in topology optimization. The problem is first solved using an exponent $\eta = 1$. The exponent is then increased by one and the problem re-solved using the previous solution as the initial guess. This process is repeated until the optimized solution contains a negligible number of inter-