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A new approach for finding the design point of nonlinear systems under random excitation

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ABSTRACT

In order to perform nonlinear stochastic dynamic analyses, the tail equivalent linearization method may be employed, which entails the adoption of the first-order reliability method. The major computational challenge posed in this method is related to finding the design point excitation for the threshold corresponding to the failure of the nonlinear system. Furthermore, finding the design point is also a sufficiently challenging problem in some importance sampling techniques. In the present paper, a new approach for finding the design point of nonlinear SDOF systems under white noise excitation is introduced, one that is identical to the mirror image excitation method for a nonlinear elastic system and yields superior results for a nonlinear hysteresis system compared to the results of mirror image excitation method. It is then extended to filtered white noise excitations and multi degree of freedom systems. Some numerical examples are given for confirming the efficiency and accuracy of the proposed method by comparing it to other methods.

1. Introduction

For assessing the safety of the structures subjected to uncertain loads such as earthquake, it is necessary to predict the response in the nonlinear range through a nonlinear stochastic dynamic analysis. Several methods can be employed to do so; such as the solution of Fokker-Plank equation, stochastic averaging method, perturbation method, moment closure method, Mont Carlo methods and equivalent linearization methods. Except for the two last methods, these methods are restricted to specific kinds of excitations or nonlinear systems and are difficult to adopt in practical applications. Simulation methods have no limitations in their application but need too much computational effort especially regarding the probability of rare events. Importance sampling methods are also used to reduce the variance in the problems involving small failure probability and increase the efficiency of the Mont Carlo method; however, these methods need the location of the design point and it is not practical where the number of uncertain problems, such as time depended dynamic problems, is too large. Au and Beck [1] proposed a subset simulation technique that is a popular method for estimating the small and very small probabilities of failure in time depended dynamic systems. Another method, namely the equivalent linearization method, has drawn much attention because of its efficiency and applicability in a wide range of nonlinear systems. In this method, the nonlinear system is replaced with a linear system such that the mean square error of the response is minimized. The classic equivalent linearization method has some shortcomings in determining the probability distribution of the response in the tail region and accurately estimating the response statistics such as the crossing rate and the first passage probability.

Because of the shortcomings inherent in the classical linearization methods, some researchers try to adopt the first order reliability method (FORM) for a new equivalent linearization method. Li and Der Kiureghian [2], Der Kiureghian [3], Franchin [4], and Koo et al. [5] have maintained the adoption of FORM for nonlinear stochastic dynamic analyses. The tail equivalent linearization method (TELM) is the most recent linearization method based on the first order reliability method (FORM) proposed by Fujimura and Der Kiureghian [6]; this method builds on and completes the previous studies, provides an accurate estimation of the response in the tail region, and overcomes the shortcomings of the classical linearization method. In this method, the stochastic excitation is discretized in terms of a finite number of standard normal random variables for solving the problem by time-invariant reliability methods. In this approach, the limit state surface of the nonlinear system is approximated using a first order reliability method by a hyper plane passing through the design point, which has the minimum distance from the limit state surface in the standard normal space.

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The design point contains valuable information about the nonlinear system, and it corresponds to the most likely realization of the stochastic input that gives rise to the failure event. It should be mentioned that the design point can be considered as a critical excitation [7] for the nonlinear system with a constraint on its input energy, duration, and frequency content.

In TELM, for each specific threshold of the nonlinear system, which corresponds to the failure event, the equivalent linear system is defined by matching the design point of the linear and nonlinear responses, and the equivalent linear system is uniquely determined in terms of its impulse response function in a nonparametric form. In fact, the major challenge in TELM is the process of finding the design point of the nonlinear system for several thresholds because it needs a great deal of computational effort. For finding the design point, gradient-based methods are employed; gradient-based methods, such as iHLRF algorithm, require the computation of the response and its gradients with respect to some realizations of random variables for a sequence of deterministic trial points until the convergence is achieved.

The time invariant reliability problems can also be solved according to the physical characteristics of the system through a simple procedure. There is a method called "Mirror Image Excitation" for finding the design point of nonlinear systems. In this method, the design point excitation of a nonlinear elastic SDOF system subjected to a Gaussian white-noise input is identical to the excitation that generates the mirror image of the free vibration response when the oscillator is released from the target threshold. This method was introduced by Koo et al. [5] for nonlinear systems. In this method, the obtained design point uses only one nonlinear dynamic analysis for a nonlinear elastic system and a few analyses for the nonlinear hysteresis systems. If this method is followed for nonlinear hysteresis systems or non-white excitations, it gives approximate results and can be used as a warm starting point for finding the exact result using gradient based structural reliability procedures.

In the present paper, it is intended to propose a new approach for finding the design point, which for nonlinear elastic SDOF systems subjected to white noise excitation, gives the same results as the mirror image excitation method and, for the nonlinear hysteresis system subjected to white noise excitation, yields a superior approximation in comparison to the mirror image excitation method. The method is then extended to filtered white noise excitations and multi degree of freedom systems; in these cases, the method yields approximate results. In this method, the direct excitation is used instead of a mirror image excitation. The procedure is established on the concept that the total mechanical energy of the system for unit input intensity is maximized in each time step when the system is subjected to the design point excitation. The adoption of this method leads to an unspecified threshold; however, by employing a simple iterative algorithm, the design point for the prescribed threshold can be found with a few iterations.

2. Design point excitation formulation

The governing equation of motion for a nonlinear single degree of freedom dynamic system subjected to stochastic input force can be considered as:

$$m\ddot{X}(t) + c\dot{X}(t) + R(X(t), \dot{X}(t)) = F(t) \tag{1}$$

where $X(t)$ denotes the time dependent displacement, m stands for the mass, c represents the viscous damping, R denotes the restoring force, and $F(t)$ is the stochastic force that is applied to the system. For solving the above equation, the stochastic force $F(t)$ is considered as a zero mean and stationary Gaussian filtered white noise process, discretized as follows [6]:

$$F(t) = \sum_{i=1}^{n} s_i(t)u_i \tag{2a}$$

where the total time interval t_n is equally spaced in n time steps at $t_i = i \times \Delta t$, and at each time step, a random pulse of amplitude u_i which has a standard normal distribution with zero mean and variance $\sigma = 2\pi S/\Delta t$ is considered, where S is the spectral density of the white noise excitation. Then these pulses are filtered by a linear filter that have impulse-response function $h_f(t)$ and are then applied to the system and [6]

$$s_i(t) = \sigma \int_{t_{i-1}}^{t_i} h_f(t-\tau)d\tau, \ \ t_{i-1} < t < t_i, \ \ i = 1, 2, \ldots, n$$

$$s_i(t) = 0, \ \ t \leqslant t_{i-1} \tag{2b}$$

In Eq. (1), it is necessary to calculate the response statistics of $X(t)$. For a specific threshold x and time t_n, the tail probability is defined as $\Pr[x < X(t_n, \mathbf{u})]$. In TELM for each threshold, the nonlinear system is replaced with an equivalent linear system such that their tail probabilities become equal. For obtaining the tail probability of the nonlinear system, the FORM approximation is used. Therefore, a limit state function $G(x, t_n, \boldsymbol{u}) = x - X(t_n, \boldsymbol{u})$ is employed, and the FORM approximation is obtained by linearizing the limit state at design point $\boldsymbol{u}^*(x, t_n)$. The design point is a point on the limit state surface that has the minimum distance from the origin and is obtained by solving the constraint optimization [6]:

$$\boldsymbol{u}^*(x, t_n) = \operatorname{argmin}\{||\boldsymbol{u}|| | G(x, t_n, \boldsymbol{u}) = 0\} \tag{3}$$

Then the limit state function is linearized at the design point:

$$G(x, t_n, \boldsymbol{u}) \approx -\nabla_u X(t_n, \boldsymbol{u}^*) \cdot (\boldsymbol{u} - \boldsymbol{u}^*) \tag{4}$$

where $\nabla_u X(t_n, \boldsymbol{u}^*)$ is the gradient vector of the limit state surface with respect to $\boldsymbol{u}$ at the design point $\boldsymbol{u}^*$. Because the limit state lies in the standard normal random variables $\boldsymbol{u}$, the distance from the origin to the linear hyper-plane 4 is called reliability index β and is given by [6]:

$$\beta(x, t_n) = \boldsymbol{\alpha}(x, t_n).\boldsymbol{u}^*(x, t_n) \tag{5}$$

where $\boldsymbol{\alpha}$ is the normalized response gradient vector [6]:

$$\boldsymbol{\alpha}(x, \ t_n) = \nabla_u X(t_n, \boldsymbol{u}^*)/||\nabla_u X(t_n, \boldsymbol{u}^*)|| \tag{6}$$

Using the design point, the gradient vector of the response is achieved by satisfying the following equation [6]:

$$\boldsymbol{a}(t_n) = \frac{x}{||\boldsymbol{u}^*(x, t_n)||} \frac{\boldsymbol{u}^*(x, t_n)}{||\boldsymbol{u}^*(x, t_n)||} \tag{7}$$

The unit impulse function h(t) of the equivalent linear system is then computed numerically by solving the following equations [6]:

$$\sum_{j=1}^{n} h(t_n - t_j)s_i(t_j)\Delta t = a_i(t_n) \tag{8}$$

Therefore, the equivalent linear system is characterized with its unit impulse response function, and through the employment of the methods of random vibration theory, the statistics of the response such as mean up crossing rate and first passage probability, defined as the complementary CDF of the maximum absolute response over a given duration, are computed.

The excitation corresponding to design point $\boldsymbol{u}^*(x, t_n)$ is called the design point excitation and can be derived using the following formula:

$$f_i^*(x, t_n) = \sum_{j=1}^{n} s_i(t_j)u_i^*(x, t_n) \tag{9}$$