ARTICLE IN PRESS

Structural Safety xxx (2016) xxx–xxx

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

Confidence-based adaptive extreme response surface for time-variant reliability analysis under random excitation

Zequn Wang Assistant Professor [a], Wei Chen Professor [b,*]

[a] Department of Mechanical Engineering-Engineering Mechanics, Michigan Technological University, Houghton, MI 49931, USA
[b] Department of Mechanical Engineering, Northwestern University, Evanston, IL 60208, USA

ARTICLE INFO

Article history:
Received 5 July 2015
Received in revised form 5 October 2016
Accepted 9 October 2016
Available online xxxx

Keywords:
Time-variant reliability
Gaussian process
Spectral decomposition
Adaptive response surface
Monte Carlo

ABSTRACT

Time-variant reliability analysis aims at revealing the time evolution of the reliability of an engineered system under time-dependent uncertainties that are best described by random processes. In practice, it is still a grand challenge to handle random process in time-variant reliability analysis due to the extremely high computational cost. In this work, a new adaptive extreme response surface (AERS) approach is proposed for time-variant reliability problems. With AERS, the dimensionality of a random process is first reduced to a set of standard normal variables and corresponding deterministic orthogonal functions based on spectral decomposition. As a result, the limit state function is reformulated as a function of only random variables and time. Next, Gaussian process (GP) models are constructed as surrogate models for predicting the value of limit state function at all discretized time nodes to approximate the extreme response surface. The accuracy of GP surrogate models is quantified by a confidence level measure and continuously improved through the sequential adaptive sampling. Using the GP surrogate models, time-dependent reliability is computed via Monte Carlo simulations (MCS). Two case studies are used to demonstrate the effectiveness of the AERS method for time-variant reliability analysis.

1. Introduction

In the past decade, various numerical methods have been developed for reliability problems without considering "time" parameter. Examples of these methods include the most probable point (MPP) based methods [1,2], the dimension reduction method (DRM) [3–5], the polynomial chaos expansion (PCE) [6–8] and the metamodeling-based methods [9–11]. In such problems, the limit state function, which indicates the state of success or failure, is a time-independent function. However, reliability generally degrades over time due to stochastic operating conditions as well as component deterioration in practical engineering applications. To model the degradation effect, the time parameter has been introduced into the limit state function either directly as an input parameter or indirectly as random process parameters. For instance, the stochastic operating load can be characterized as a random process with an explicit time parameter. Even though significant progresses have been made in time-independent cases, ensuring a high level of time-variant reliability during a product life cycle remains challenging in practical engineering applications, as it requires efficient ways for handling time-dependent reliability assessments.

In time-variant reliability analysis, uncertainties are propagated to the response of engineered system to quantitatively assess the probability that the system fulfills its intended function over a specified time period. Uncertainties may come from manufacturing processes, material properties, and operation conditions that can be characterized by random variables and random processes. As a result, the limit state function of system performance is a complex random process such that accurately assessing the time-variant reliability function using simulation-based methods, e.g., Monte Carlo simulation, [12,13] is computationally prohibitive.

Various time-variant reliability methods were developed mainly including the extreme value based approaches [14–18] and the first-passage based approaches [19–23]. The key to extreme performance approaches is to identify the extreme value of a limit state function, and then quantify the uncertainty of the extreme values to approximate time-variant reliability. One example is the nested extreme response surface (NERS) framework [16] which was developed to construct a response surface over the random input space for predicting the time that leads to the extreme response of interest. With the time response surface, time-variant

* Corresponding author.
E-mail addresses: zequnw@mtu.edu (Z. Wang), weichen@northwestern.edu (W. Chen).

http://dx.doi.org/10.1016/j.strusafe.2016.10.001
0167-4730/© 2016 Published by Elsevier Ltd.

Please cite this article in press as: Wang Z, Chen W. Confidence-based adaptive extreme response surface for time-variant reliability analysis under random excitation. Struct Saf (2016), http://dx.doi.org/10.1016/j.strusafe.2016.10.001

Nomenclature

R	reliability	I_f	indicator function
Φ	standard Gaussian cumulative distribution function	P_f	probability of failure
β_t	target reliability index	ν^+	out-crossing rate
EI	expected improvement	$\lambda(t)$	out-crossing rate
$f_x(x)$	probability density function	$\mathbf{R}$	correlation matrix
$f(\cdot\|\cdot)$	conditional probability density function or likelihood function	**Cov**	covariance matrix
p_f	probability of failure	$\mathbf{M}$	GP model
		T_f	time to failure

reliability problems are converted to time-independent counterparts, which enables the integration of existing advanced static reliability tools into time-variant reliability approximation. Though NERS shows high efficiency, it lacks the ability of handling random process parameters. In order to tackle random processes as inputs, Mourelatos [18] decomposed the random processes using Eigen analysis and computed the conditional reliability for subsets by the first-order reliability method (FORM). The time-variant probability was approximated by the integration of the conditional probabilities over all subsets. The integration of conditional probabilities, however, is computationally expensive requiring numerous FORM calculations and may induce errors for highly nonlinear cases.

The first-passage based approaches were developed based on the "out-crossing events" during a specific time period, a concept first introduced by Rice [23]. A failure occurs if the performance function exceeds the upper bound or falls below the lower bound of the safety threshold. By assuming the independency of out-crossing events, the time-variant reliability is assessed by the integration of the out-crossing rate that represents the rate of reliability change with respect to time. To obtain the out-crossing rate, the PHI2 approach [22] was developed to track the rate in the U-space using FORM, and then to approximate the out-crossing rates based on the reliability indices of two successive time nodes. Although the FORM is seamlessly integrated, the accuracy of out-crossing rate based methods is limited. In addition, the FORM method generally requires accurate sensitivity information of limit state function with respect to random variables, which is often not available in practical engineering applications. Recently, surrogate model-based methods [24–28] have been developed to alleviate the computational burden of time-dependent reliability assessment. Most of existing studies [26,27] do not incorporate the characteristics of random system variables into adaptive updating scheme, which may constraint the rapid convergence of probability approximations. Another limitation shared by these methods lies in the lack of capability to quantitatively measure the fidelity of surrogate models and the accuracy level of reliability approximations. With existing methods, it is difficult to develop appropriate stopping rules for Kriging updating process. In the literature, a number of studies [29–32] have been done to adaptive update surrogate models based on prior information. The efficient global optimization [29] proposed the expected improvement measure to locate new point for updating Kriging models, which shows promising results in deterministic problems. For probability analysis methods [30–32] based on adaptive surrogate models, it still remains challenging to quantitatively measure and effectively enhance the fidelity of surrogate models.

In this work, we propose a time-variant reliability method using adaptive extreme response surfaces that can handle both random variables and random processes as input uncertainty. Using spectral decomposition, random processes are first discretized and represented by a set of independent standard normal variables and deterministic functions. Gaussian process models are then used to build response surfaces for predicting the limit state function at all discretized time nodes. A confidence level is employed to measure the fidelity of GP models and an adaptive sampling scheme is utilized to locate the sample points for updating the GP models. The extreme value of time-dependent function for a given time period is thus approximated by the GP models for instantaneous time nodes within a specified time interval, and MCS are used to approximate the time-variant reliability. AERS possesses several advantages over traditional time-variant reliability methods. First, AERS is a sensitivity-free method, which can reduce the computational cost as well as enhance the robustness of time-variant reliability analysis. Secondly, we can assess the accuracy of time-variant reliability approximation by a quantitative confidence-based measure, which indicates whether an extreme response surface needs further updating. An adaptive updating scheme is used to wisely allocate the computational resource by selecting the most important samples for updating the response surface models.

The paper is arranged as follows. Section 2 introduces existing methods for time-variant reliability analysis. Section 3 describes the proposed adaptive extreme response surface approach including the treatment of input random processes by spectral decomposition, Gaussian processing modeling, and the confidence-based adaptive sampling technique. Two case studies are used to demonstrate the effectiveness of the developed methodology in Section 4.

2. Review of time-variant reliability analysis

In this section, we provide a general definition of time-variant reliability and conduct a literature review of existing methodologies.

2.1. Definition of time-variant reliability

In engineering applications, system or component failure occurs if a time-dependent performance function goes beyond its failure threshold. Due to the existence of uncertainties such as those coming from material properties, manufacturing variations, and loading conditions, the preformation function of an engineered system is often a complex stochastic process. With the uncertainties characterized by a set of random variables and random processes, a general limit state function is expressed as $g(\mathbf{X}, \mathbf{Y}(t), t)$, where $\mathbf{X} = [X_1, X_2, \ldots, X_r]$ is a vector of r random variables, $\mathbf{Y}(t) = [Y_1(t), Y_2(t), \ldots, Y_m(t)]$ is a vector of m input random processes with respect to time t. Within a specified time period $[0,T_L]$, a failure occurs for a specified realization of $(\mathbf{X}, \mathbf{Y}(t))$ if there is a time node t in $[0,T_L]$ such that the limit state value goes beyond the safety margin, which is described by

$$\exists t \in [0, T_L], g(\mathbf{X}, \mathbf{Y}(t), t) > 0. \tag{1}$$

The time-variant probability of failure over a time period $(0, T)$ $P_f(0, T)$, which is generally a monotonic increasing function with time T, is defined as