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Multiple response surfaces method with advanced classification of samples for structural failure function fitting

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ABSTRACT

The current response surface methods based on classifier usually fail to classify all samples correctly, thus neglect the effects of the misclassified samples on the fitting function. To overcome this issue, an improved multiple response surfaces method is proposed. It is mainly based on the techniques of sector division and correct classification of samples. The main steps are: (1) compute a normalized inner product coefficient between the closest sample to the origins and any other one, and sort samples by the coefficient values; (2) select a reasonable number of sorted samples (i.e. range of normalized inner product coefficient) for each sector to assure that the samples in the sector can be classified correctly; (3) divide the overall space into multiple sectors based on such ranges and execute an approximation sector by sector based on support vector machines. A main merit of this method is that it can approximate implicit failure functions well as the number of samples is large enough due to the features of the correct classification of all samples. In addition, it can be applied to both single failure functions and multiple failure functions (explicit ones and enveloped ones). Numerical examples show that the proposed method can achieve a good fitting of implicit failure functions, and the reliability results are accurate, too.

1. Introduction

For a mechanical structural system with uncertainty, the estimation of its reliability provides valuable information. For a simple structure, the failure function would be explicit and the reliability analysis can be performed effectively by the first order reliability method (FORM), the second order reliability method (SORM) or Monte Carlo simulation (MCS). However, for a large and complex structure, such failure function is usually implicit, complex (e.g. piecewise and nonlinear) and in a high-dimensional space. In this case, the conventional FORM, SORM and MCS would be less efficient or accurate.

To overcome these difficulties, Der Kiureghian et al. [1,2] proposed a search algorithm and strategies for finding the multiple design points, and Katafygiotis [3,4] et al. developed a spherical subset simulation method for solving high-dimensional reliability problems. Numerical examples indicate that these measures need no surrogate model and can increase the efficiency and accuracy of reliability analysis.

Another strategy for dealing with the difficulties is to obtain an explicit approximation (i.e. surrogate model) of the implicit failure function of the structure before performing a reliability estimation. As a useful tool for modelling and analyzing, the response surface method (RSM) has attracted significant attention due to its computational efficiency and convenience in combination with common software.

Faravelli and Bigi [5,6] discussed a stochastic finite element method based on response surface approximation to analyze the reliability of structural and mechanical systems whose geometrical and material properties have spatial random variability. Bucher and Bourgund [7] studied a new adaptive interpolation scheme of updating polynomial to increase the efficiency and accuracy of the response surface method in reliability calculations. Quite a number of measures have been proposed to improve the efficiency and accuracy of the conventional response surface method. These improvements mainly concern approaches that use more complex function models, such as complete quadratic polynomials [8], higher-order polynomials [9], adaptive models with selected terms [10], and artificial neural networks (ANN) [11–14]; and approaches

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that use efficient techniques to allow the approximation to be closer to the limit state function at the design point, such as the weighted regression method [15,16] and the experimental points moving schemes [17,18]. For a low dimensional case (e.g. less than 4 input variables), it is stated that such improvements are quite capable of approximating failure functions of structural systems [19,20]. However, the fitting accuracy would be largely affected by the number of sample points, and the generalization error would also increase largely as the number of input variables increases, because these improvements are mainly based on the principle of empirical risk minimization (i.e. fitting residual minimization) [21].

Based on the statistical learning theory, an optimal way to minimize the generalization error of a learning machine is following the principle of structural risk minimization for a high-dimensional case. Support vector machines (SVM) are one of the best options to follow this principle because they only use the support vectors rather than any other samples to fit a function. With this unique property of SVM, an accurate fitting function and reliability results can often be achieved [22–24].

As mentioned earlier, the real failure function of a large structural system would usually be of a complex structure and high-dimensional. It is clear that a single response surface, whether it is based on SVM, or polynomials, or ANN, cannot approximate the real failure function well in this case. Thus, a reasonable way is adopting multiple response surfaces to obtain an accurate approximation [25].

Recently, the multiple response surfaces method has attracted attention in slope reliability analysis [26,27]. In these applications, each possible failure mode (i.e. slip surface) of the slope can be identified by the Bishop method before using a quadratic polynomial model to perform an approximation, and thus multiple failure functions can be obtained one (failure mode) by one. Following this way, an integral reliability can be estimated easily if the assumption of a series system is adopted. Unlike the reliability problems in a slope system, each failure mode of a large structural system is difficult to be identified and is generally unknown before using RSM to perform an approximation. Thus, Neves et al. [28] recommended to regard the system failure function as an integral enveloped one with complex and high-dimensional characteristics and use RSM to approximate the enveloped function directly.

A simple measure to achieve a good approximation of a complex function is dividing the overall space into many hypercubes based on the divided ranges of each variable, and obtaining an approximation to the complex function in each hypercube (see [29]). Note that such measure cannot be applied well to a high-dimensional case because the needed number of hypercubes (i.e. response surfaces) would increase exponentially as the number of variables increases, resulting in a time-consuming computation.

To reduce the computational cost, Mahadevan and Shi [30] proposed an approach to approximate the real failure function with multiple hyperplanes, and a way to calculate the failure probability through the union or intersection of the failure domains corresponding to each segment. Liu and Lv [31] proposed a similar approach for response surfaces combination in reliability analysis. One of the main advantages of these approaches is that they can be useful for both component and system reliability problems. However, it is difficult to determine whether a failure domain defined by the corresponding response surface contributes through a union operation or an intersection operation to the overall failure domain for reliability estimation, because such operation may vary largely in different domains when the real failure function is of a complex nature and high-dimensional. Thus, the multiple response surfaces method still needs to be improved further in efficiency and accuracy.

Herein, we propose a method for correct classification of samples to fulfill this demand, which is mainly based on the techniques of sector divisions of the overall high-dimensional space and SVM. The proposed method as well as an iterative algorithm is used to achieve a converged solution in function fitting. The computational efficiency and accuracy are also studied for the proposed method.

2. Classifying models

2.1. Short review of SVM

This section is devoted to a short description of the SVM method of classification. More details can be found in [32,33].

Given is a set of N training samples $(\boldsymbol{x}_i, h_i)$ $(i = 1,2,\ldots,N)$ with binary outputs $h \in \{+1, -1\}$ corresponding to the two classes. Assume that the two classes of training samples are linearly separable. Then, two parallel hyperplanes H_1 and H_2 can be selected to achieve separation, as shown in Fig. 1. Thus, the margin width between these two hyperplanes is $2/\|\boldsymbol{w}\|$.

Based on the SVM theory, the optimal hyperplane H is the one that represents maximum margin width and lies halfway between the hyperplanes H_1 and H_2, which is expressed as the optimization problem: maximize $2/\|\boldsymbol{w}\|$ and subject to $|\boldsymbol{w}\boldsymbol{x}_i + b| \geqslant 1$ for all $\boldsymbol{x}_i$. Then, the optimum linear classifier can be obtained by solving this optimization problem, and it is given by

$$G(\boldsymbol{x}) = \mathrm{sgn}\left[\sum_{i=1}^{n} \alpha_i^* h_i(\boldsymbol{x}_i \cdot \boldsymbol{x}) + b^*\right] \tag{1}$$

where $\mathrm{sgn}(\cdot)$ means the sign function; $(\boldsymbol{x}_i \cdot \boldsymbol{x})$ means the inner product operation; α_i^* and b^* are two relevant parameters to define the optimum linear classifier. For most samples, $\alpha_i^* = 0$. By comparison, for support vectors, $\alpha_i^* \neq 0$.

2.2. Quadratic function model

Let $\boldsymbol{x}$ denote the normalized vector of random variables, $\boldsymbol{x} = [x_1, x_2, \ldots, x_n]$, where n is the number of variables. For a nonlinear function fitting, a model of quadratic polynomials without cross terms is often used. In this paper, such model is also selected and a corresponding transformation from $\boldsymbol{x}$ space to $\boldsymbol{z}$ space is given by

$$\boldsymbol{z} = [x_1, x_2, \ldots, x_n, x_1^2, x_2^2, \ldots, x_n^2] \tag{2}$$

Thus, in the $\boldsymbol{z}$ space, the optimum linear classifier is expressed by

$$G(\boldsymbol{z}) = \mathrm{sgn}\left[\sum_{i=1}^{n} \alpha_i^* h_i(\boldsymbol{z}_i \cdot \boldsymbol{z}) + b^*\right] \tag{3}$$

It is known that such a classifier is actually a quadratic support vector machine (QSVM) in $\boldsymbol{x}$ space.

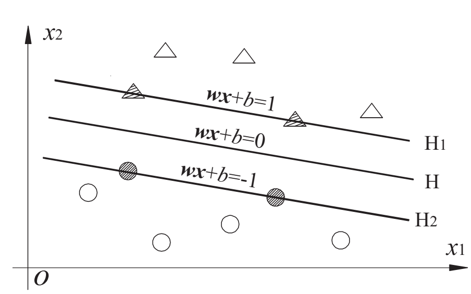

Fig. 1. Optimal hyperplane for linearly separable case.