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Network reliability analysis with link and nodal weights and auxiliary nodes

Roberto Guidotti [a,*], Paolo Gardoni [a], Yuguo Chen [b]

[a] Department of Civil and Environmental Engineering, MAE Center: Creating a Multi-hazard Approach to Engineering, University of Illinois at Urbana-Champaign, 205 North Mathews Ave., Urbana, IL 61801-2352, USA
[b] Department of Statistics, University of Illinois at Urbana-Champaign, 725 South Wright St., Champaign, IL 61820, USA

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ABSTRACT

Networks are omnipresent, with examples in many different fields, from biological networks (such as the nervous and cardiovascular system) to physical networks (such as roadways, railways, and electrical power and water supply systems) to technological networks (such as the World Wide Web) and social network (such as the community network among people or animals). This paper proposes a novel probabilistic methodology to quantify the network reliability based on existing (diameter and efficiency) and new (eccentricity and heterogeneity) measures of connectivity that incorporate link and nodal weights and auxiliary nodes. Nodal and link weights are introduced to take into account the importance of the components in the topology-based network model. Unweighted auxiliary nodes, locally refining the network model, allow one to capture the complexity of the connections between weighted end-nodes. The formulation presented in this paper is general and applicable to networks in different fields. The paper illustrates the implementation of the proposed formulation considering a transportation network subject to seismic excitation.

1. Introduction

A network is generally defined as a system of interconnected elements or nodes. Networks can be found in many different fields. For example, in the human body, we find networks in the nervous system, with neural networks connecting neurons with synapses, and the cardiovascular system, with a complex net of blood vessels connecting other systems, such as the respiratory, digestive or muscular system. Examples of physical networks are those that allow the transportation of people or resources, such as roadways or railways, electrical power and water supply systems. Other examples are technological networks, such as the World Wide Web, and social networks, such as those among people or animals. While in different fields, networks can often be modeled using a common formulation. In recent years, many studies in statistics dealt with technological networks and social networks. They focused mainly on the state of the network, evaluating the impact of a random or targeted attack [1–8].

Recent developments on the reliability of networks were made in the context of physical networks, like transportation networks, pipelines and other lifelines, vulnerable to seismic events [9–13]. Physical networks connecting communities and critical facilities play indeed a strategic rule in the aftermath of extreme natural events like large earthquakes, allowing for rescue and recovery efforts. As pointed out by a review paper from Frangopol and Bocchini [14], researches in this field have followed different aspects, including: (i) the interaction of bridges in a spatially distributed system, (e.g., [15–18]); (ii) the variability in time of the characteristic of a bridge network, (e.g., [19–22]); (iii) the interdependency among different networks and the cascading failures (e.g., [23–27]); (iv) the economic constraints in the life-cycle analysis of the network (e.g., [28,29]). From a computational point of view, Monte Carlo simulation methods have been widely used for the study of system reliability (e.g., [30–33]). These methods have been applied to transportation network reliability (e.g., [17,34–37]). Monte Carlo simulation results are often a benchmark in the development and application of non-sampling methods, such as the matrix-based system reliability (MSR) method for transportation networks [16,18,22].

Most of the current literature focuses on the failure of single links of the network, and the concept of network failure in topology-based models is typically defined as the loss of connection after a disturbance between some nodes and the rest of the network. In this paper, the links are the only elements subject to failure. This assumption is often verified in physical networks

* Corresponding author.
E-mail address: guidott2@illinois.edu (R. Guidotti).

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subject to hazardous event, where, for example, bridges, pipelines and other lifelines are the most vulnerable elements. The network reliability based on deterministic values of the measures of connectivity currently adopted in the literature, such as the diameter and the efficiency, presents however some limitations. First, the measures of network connectivity have a variability that needs to be considered for a better estimate of the reliability and service reliability of networks. Second, while current approaches in topology-based network models typically include weights to capture the different importance of links, they do not take into account the different importance of the nodes. In the case of transportation networks, for example, one could assign a different importance to each node proportionally to the population served by the node. However, approaches based on current definition of diameter and efficiency do not permit to make such distinction. Third, current approaches place a single link between two nodes. However, often networks have a complex system of in series and in parallel sub-links between two end-nodes. Using a single link between two nodes does not allow the proper evaluation of the loss of capacity of the connection in case of a disturbance. This is because a single link can typically be either functioning or not. However, in reality a link could experience only a partial loss of capacity due to failure of some, but not all, of the sub-links connecting the two end nodes. As a result, the characterization of the network reliability is incomplete and potentially misleading.

This paper addresses the above limitations by making the following contributions. First, beside diameter and efficiency, two novel measures of connectivity, namely eccentricity and heterogeneity, are introduced to take into account the variability in the connection measures between two generic nodes in the network. Second, nodal weights are introduced in addition to link weights to take into account the importance of the components of the network and to provide, indirectly, a simplified measure of flow. Third, unweighted auxiliary nodes capture the complexity of the connections between end-nodes. They help to represent the complex network topology between two nodes, allowing a proper evaluation of the loss of capacity of the connection between the end-nodes. The introduction of the proposed nodal weights and auxiliary nodes implies a modification of the adopted measures of network connectivity. Overall, we propose a probabilistic formulation to assess the network reliability of networks by investigating the proposed connectivity measures that include nodal weights and auxiliary nodes. The formulation is probabilistic because it allows for the probabilistic characterization of the reliability of a network, based on the statistics of the adopted four connectivity measures in the post-event scenario. The formulation is illustrated by an example transportation network that connects 8 cities by highways with 12 bridges subject to seismic excitations. This application clearly shows the value of the proposed formulation for a quantitative evaluation of the reliability of the considered network.

The paper is organized in the following sections: Section 2 reviews the current formulations; Section 3 presents the novel contributions including adopted measures of connectivity, nodal weights and auxiliary nodes; Section 4 describes the probabilistic algorithms to simulate the effects of a disturbance on the network; and, finally, Section 5 illustrates the proposed formulation using an example transportation network.

2. Current formulations of the network characteristics

2.1. Network representation: adjacency, weight and network table

As presented in Watts and Strogatz [38], a general undirected network can be defined by n nodes or vertexes and m links or edges connecting the nodes. The network is then represented by an $n \times n$ adjacency table $\mathbf{A} = [a_{ij}]$, where a_{ij} $(i \neq j)$ is either 1, if there is a link between nodes i and j, or 0 otherwise, and $a_{ii} = 0$. It is common, especially for physical networks, to weight the adjacency table with an $n \times n$ link weight table $\mathbf{W}_L = [w_{L,ij}]$, where the weights $w_{L,ij}$ $(i,j = 1,\dots,n)$ are used to capture a characteristic of interest of the link between nodes i and j. For example, in the case of a transportation network, $w_{L,ij}$ could represent the distance or the travel time between the two end-nodes i and j. In this case, the link-weighted network is represented by an $n \times n$ network table $\mathbf{N} = [n_{ij}]$ with element $n_{ij} = a_{ij} \times w_{L,ij}$ $(i,j = 1,\dots,n)$.

2.2. Measures of connectivity: diameter and efficiency of the network

Networks can be distinguished based on measures of connectivity. Two end-nodes are connected if there is at least one path between them with a finite number of links. There are two typical measures of network connectivity in the literature: the diameter (or characteristic path length) δ and the efficiency η. Both can be defined to describe the connectivity of a specific node to the other nodes in the network (nodal diameter and nodal efficiency) or to describe the overall network connectivity as an average of all of the nodal connectivity (global diameter and global efficiency).

The nodal diameter δ_i is the average length of the shortest path between node i and the rest of the network:

$$\delta_i = \frac{1}{(n-1)} \sum_{\substack{j=1 \\ j \neq i}}^{n} d_{ij}, \tag{1}$$

where d_{ij} $(i,j = 1,\dots,n)$ is the length of the shortest path between nodes i and j, i.e., the smallest sum of the link weights (e.g., distances) considering all the possible paths in the network between node i and node j, [39]. The quantity in Eq. (1) can then be standardized by dividing it by the optimal nodal diameter $\delta_{i,opt}$ that corresponds to an ideal network with a direct (single) link between each pair of nodes (complete graph): $\bar{\delta}_i = \delta_i/\delta_{i,opt}$. As a note about the nomenclature used in this paper, we use the word "standardized" to indicate that the considered metric is divided by a reference value. To avoid a possible confusion, we do not use the word "normalized" here and keep this word to indicate a transformation that makes a variable to follow a Normal distribution. In the case of an unweighted graph, $\delta_{i,opt} = 1$. The standardized nodal diameter $\bar{\delta}_i$ ranges from 1 to $+\infty$. It is equal to 1 when $\delta_i = \delta_{i,opt}$, and higher values of $\bar{\delta}_i$ indicate some loss of connectivity with respect to the optimal case. In the case node i is disconnected from node j, it is not possible to find a path of finite length between the two nodes and $d_{ij} = \infty$. As a result, δ_i and $\bar{\delta}_i$ are equal to ∞. However, we do not have information on the extension of loss of connectivity, in other words, whether just one node or a larger number of nodes lost their connection with node i.

To address this issue, Latora and Marchiori [39], introduced the efficiency η. The nodal efficiency η_i is defined as the average of the inverse of the shortest path between node i and the other nodes of the network:

$$\eta_i = \frac{1}{(n-1)} \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{1}{d_{ij}} = \frac{1}{(n-1)} \sum_{\substack{j=1 \\ j \neq i}}^{n} h_{ij}, \tag{2}$$

where $h_{ij} = 1/d_{ij}$ for $i \neq j$ and $h_{ij} = 0$ otherwise. Unlike δ_i, η_i provides information also on the extension of the loss of connectivity. When node i is disconnected from node j, $d_{ij} = \infty$ and, as a result, $h_{ij} = 0$. A larger number of $h_{ij} = 0$ in the summation in Eq. (2) reflects a larger number of disconnections, resulting in a lower value of η_i. The value of η_i is between 0 (no links between node i and the other nodes) and 1 (in the case of a complete graph.) As for the nodal diameter, η_i can