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Interval reliability analysis under the specification of statistical information on the input variables

Jorge E. Hurtado *, Diego A. Alvarez, Jairo A. Paredes

Universidad Nacional de Colombia, Apartado 127, Manizales, Colombia

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ABSTRACT

Structural reliability analysis should often face the problem that there is uncertainty about the probabilistic specification of the input random variables implied in a specific problem. In the poorest information case only the knowledge of intervals of fluctuation is available, while the opposite situation is that of a full specification of the joint distribution function of the variables. Parametric or non parametric probability boxes, arising from basic statistical tools such as confidence intervals or Kolmogorov–Smirnov distance, are intermediate situations. In any case, random or fixed intervals emerge for describing eitheistribution parameters under epistemic uncertainty. A crude Monte Carlo simulation for assessing the corresponding interval of the failure probability requires the solution of a double optimization problem for each probability level of the limit state function. This, obviously, implies a prohibitive computational labor. In this paper a method for drastically simplifying this task is proposed. The method exploits the ordering statistics representation property of the reliability plot, which is shown to approximately obey an orthogonal hyperbolic pattern. Accordingly a two-level FORM approach with relaxed accuracy constraints is used to derive the polar vectors for building two plots, one for the input variable space and the second to the epistemic uncertainty space. Using a variety of examples, it is demonstrated that the extrema of the failure probability are contained amongst the samples located in specific sectors of the upper level plot as indicated by the hyperbolae. This means that, after solving the two-level FORM problem, it suffices to calculate the failure probability (or the reliability index) for a small number of mean samples thus selected. Obviously, the method yields the same reliability interval estimates as the crude two-level Monte Carlo because its samples are underlying in it.

1. Introduction

The most accepted definition of the reliability P_s of a structural or mechanical system in the framework of classical probability theory is [1]

$$P_s = 1 - P_f \tag{1}$$

where P_f is the probability mass in a failure domain $\mathcal{F}$ of the space of the basic random variables $\boldsymbol{x}$ of d dimensions, determined by a limit state function $g(\boldsymbol{x})$, i.e. $\mathcal{F} = \{\boldsymbol{x} : \boldsymbol{g}(\boldsymbol{x}) \leqslant \boldsymbol{0}\}$. The failure probability is defined as

$$P_f = \int_{\mathcal{F}} p_{\boldsymbol{x}}(\boldsymbol{x}) \mathrm{d}\boldsymbol{x} \tag{2}$$

where $p_{\boldsymbol{x}}(\boldsymbol{x})$ is the joint density function of the basic variables. An alternative meaningful measure of the reliability is the geometric index [2]

$$\beta = \Phi^{-1}(1 - P_f) \tag{3}$$

where Φ is the standard Gaussian distribution function, or the generalized index [3].

This classical approach to solve the reliability problem presupposes the knowledge of the joint density function $p_{\boldsymbol{x}}(\boldsymbol{x})$. But at this point epistemic uncertainties emerge. In fact, not only there is rarely sufficient knowledge on the mutual dependencies among the basic random variables [4] but also there is sometimes uncertainty either about the models used for marginal distributions or about the parameters of a well selected model.

This problem has fostered the research on methods for estimating the effect of these uncertainties on the failure probability. For instance, in [5] the Total Probability Theorem is applied to derive unconditional estimates of the failure probability upon consideration of the uncertainty of seismic parameters, while in [6] a

* Corresponding author.
E-mail address: jehurtadog@unal.edu.co (J.E. Hurtado).

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method equivalent to the application of the theorem on parameter uncertainty is proposed. This method is applied in particular for estimating the reliability index under some kinds on probabilistic uncertainties in [7]. In [8–10], the first-order reliability method (FORM) is extended for variables grouped in two sets, one of variables defined with fully specified probability distributions, and another with variables defined only up to intervals of fluctuation, while in [11] a numerical method for combining variables with specified probability distributions or intervals is proposed. In [12,13] the calculation of reliability intervals of systems is made on the basis of a FORM approach and in [14] a numerical method for the case of a single interval-defined parameter per input distribution, based on the monotonicity of the distribution functions, was developed. In the field of Monte Carlo simulation methods, an interval finite-element approach for linear structural analysis and a simulation method for calculating the histogram and the confidence intervals of the failure probability is proposed in [15]. On the other hand, in [16] the Importance Sampling concept is applied to the estimation of the reliability interval.

A different route is taken in the approaches based on the random set theory [17–21]. In [22], for instance, it is proposed to construct probability boxes on the basis of Tchebycheff's inequality for the assessment of reliability bounds with the aim of being consistent with the scarce randomness information, while in [23] the general problem of combining different types of variables with copula-type of dependencies is tackled under the general framework of such sets. Finally, the problem of uncertain distribution has been also modeled as a fuzzy-random problem in [24].

Also, some proposals for estimating the reliability intervals have made use of the tools of classical interval analysis [25,26]. Important formulas for estimating the reliability intervals of series, parallel and series–parallel systems have been derived in [27]. Nevertheless, for estimating reliability intervals of each component of such systems, interval analysis is of limited applicability, due to the inherent limitations of interval arithmetic for complex problems [28]. These problems are avoided when using a proposal for interval and fuzzy analysis published recently [29–31], based on the technique of the *Reliability Plot* (RP) introduced in [32]. The ideas and concepts reported in these references are combined and improved herein, as explained along the paper. Since the method proposed herein is a further extension of the RP methodology, some comments on its roots are in order.

The RP is based on the fact that the classical hyperplane approximation in structural reliability [2,3,1], from which the FORM and SORM methods stem, is highly useful for a simulation approach due to its geometric potential, because the normal vector to the hyperplane posed by the FORM approximation reveals the topological clustering of the safe and failure samples given by simple Monte Carlo Simulation (MCS) or other simulation methods in static problems. This is valid despite the shortcomings of FORM in the strict probabilistic sense [33–36]. This means that the samples useful for improving the FORM can be easily selected from a two-dimensional plot, thus yielding the same solution as MCS with a minimal computation effort additional to that needed to derive the FORM vector [32,31,23]. Notice that, under the ignorance of the actual probability distribution of function $g(\boldsymbol{x})$, an intense MCS is normally taken as the reference for the exact value of the probability of failure.

In other words, the RP construction is based on some valuable geometric properties of the classical hyperplane construction of structural reliability analysis that remain unnoticed for several years. They are the following:

- In the standard $\boldsymbol{u}$-space of standard Gaussian variables there is an implicit system of two polar coordinates equipped with a spherical symmetric probability measure.
- The mapping of the Gaussian samples to the space constituted by these two coordinates allows a drastic dimensionality reduction because they are rigorously independent in the probabilistic sense.
- Moreover, the mapping to this space contains an implicit representation of the evolution of $g(\boldsymbol{u})$.

These properties are demonstrated and exploited herein to assess the interval of fluctuation of the failure probability, i.e. $[\underline{P}_f, \overline{P}_f]$, by construing an additional space to the given standard Gaussian one, which is made of truncated Gaussian variables for the mapping of the fluctuation intervals of the input distributions. The main advantage of the proposed method is that it can be applied to different models of uncertain input variables, the most common of which are the following:

- Intervals.
- Conventional probability distributions.
- Parametric probability boxes, i.e. probability distributions with parameters specified as intervals. An important particular case is that of *confidence intervals*, in which the distributions of some parameters are known.
- Non-parametric probability boxes, i.e. those specified with lower and upper bounds of empirical probability distributions, such as those arising in applying the well-known Kolmogorov–Smirnov test or those construed with, e.g., Tchebychev inequality [22].

The next sections are devoted to an explanation of the method in detail. First, a formal exposition on ways of specifying the uncertainty on variable description is given. In this regard, it is important to state that the paper does not delve into the disputed issue on the adequate modelling of variables under scarce empirical information but its concern is exclusively the numerical assessment of the interval of fluctuation of the failure probability under variables modeled as described in the above list. After such exposition, the essentials of the RP technique, together with some new insights on its capabilities, is provided. This is followed by a presentation of the proposed approach, which is then illustrated with four examples that correspond to different cases in structural and geotechnical analyses. The paper ends with some conclusions.

2. Problem formulation

In the structural reliability field, it is important to recall that, in many cases, statistical information on some or all the input random variables implied in a specific problem is specifically collected. This is a normal situation in soil mechanics and also occurs in structural building construction as a consequence of quality control measures, health monitoring, etc. But it is also true that in other situations the empirical information about one or more important variables is rather scarce, as is the case of those given by rare natural phenomena.

On the basis of n statistical data, several estimates can be drawn, among which the most common are the empirical mean of a random variable x,

$$m = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{4}$$

which is an unbiased estimate of the actual mean μ and the empirical variance

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - m)^2 \tag{5}$$