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Probabilistic interval geometrically nonlinear analysis for structures

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ABSTRACT

This paper presents a new unified Chebyshev surrogate model based hybrid uncertainty analysis method for robustly assessing geometrically nonlinear responses of engineering structures involving both random and interval uncertainties. In this proposed approach, Chebyshev response surface strategy combined with finite element framework is developed to model the nonlinear relationships between the uncertain structural parameters and the corresponding system responses. A comprehensive computational analysis framework, namely generalized unified interval stochastic sampling, is devised to furnish the statistical features, including means, standard deviations, probability density functions and cumulative distribution functions, of the lower and upper bounds of the nonlinear random interval structural behaviours. The applicability and notable performance of the presented approach are elucidated with the help of two practically motivated examples.

1. Introduction

Structural analysis quantifies the responses such as displacement and stress state on physical structures due to external or internal loads. Such responses are directly related to the geometric and material properties of the structures. In engineering applications, these properties are usually linearized by engineers and designers in order to simplify the problems as nonlinear analysis procedures are more complex and time consuming. However, typical linear analysis becomes incompetent for situations involving the implementation of new materials and when the structural safety is rigorously required. Instead, nonlinear structural analysis is more rational and should be performed so that a more optimized structure can be obtained [1,2].

Besides various types of nonlinearity involved in structural analysis, uncertainty and randomness are also inherent in the geometric and material properties, which influence significantly the structural performance. Possible sources of uncertainties are including variational material properties caused by the manufacturing defects, arbitrariness intrinsically embedded within material attribute, inconsistent data measurement of experiments due to human errors, as well as the mutations of operating conditions due to the random nature of the environment [3–5]. The significance of considering uncertainties within structural analysis has been well acknowledged across many engineering applications. Therefore, it is vital and essential to consider the impacts of uncertainties of system parameters in both structural serviceability and strength limit analysis and design [6,7].

Depending on the nature of uncertain variables, three major analysis frameworks have been extensively developed which are probabilistic/stochastic, non-probabilistic and hybrid approaches. In the first analysis category, by implementing the theory of probability and statistics, uncertain system parameters are modelled as either random variables or random fields characterized by the corresponding distribution functions [8,9]. This approach enjoys the priority when sufficient data is available for the uncertain variables as it is capable of providing all desired statistical information of structural outputs. The second category, namely non-probabilistic approach [10,11], addresses situations when stochastic modelling encountered difficulties and insufficiency in data collection. This approach broadly includes the format of fuzzy approach [12], interval analysis [13–15], convex model with ellipsoidal uncertainties [16,17] and info-gap analysis [18] etc. In the third category, any combinations of the methods from the first and second categories can be incorporated. The hybrid approach offers a more universal analysis framework by tolerating mixture of distinctive uncertainty models, such that a more superior uncertainty analysis can be conducted basing on the first-hand information on system uncertainties [19–20].

The task of accurately assess the nonlinear behaviour of a structure is a non-trivial one, and it becomes even more challenging when taking into consideration of the uncertainties of structural parameters due to the lack of a priori knowledge. Numerical

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modelling techniques such as finite element method (FEM) have long been implemented to handle complex nonlinear problems. Despite the rapid improvement of computational power and continuous development of efficient solving algorithms, the computational cost that takes to solve complex and nonlinear structural problems is still excessive [21]. In such case, traditional direct sampling approaches for uncertain analysis cannot be implemented as obtaining the uncertain outputs is time-consuming and will be valueless for engineering practise. In the light of current situation, alternative approaches have been acknowledged to improve the computational efficiency. Advanced sampling schemes have been developed in order to cut down the sample size without sacrificing the accuracy, for instance, the importance sampling approach [22], subset simulation method [23], the line sampling strategy [24] etc. Although, the sample size in these approaches has been reduced compared with Monte Carlo simulation method, it is still important to find more efficient methods especially for the circumstance that complex finite element models are involved [25] or multiple types of uncertainties are encountered. Such deficiency highly limits the advancement in nonlinear uncertainty analysis, to the authors' knowledge only a few researchers have addressed this class of problems, among which mostly have focused on single type of uncertainties [26,27]. However, uncertainties are miscellaneous in modern engineering systems due to the obligations to satisfy numerous public demands. Surrogate model strategy [28,29] constitutes an approximated explicitly formulated model of the performance function, which is usually implicit, by using a set of observations in the identified area of interest on the performance function. This approach provides a computationally inexpensive way to predict the structural responses with sufficient accuracy. Consequently, computational cost can be reduced within the deterministic function and uncertainty analysis of complex structures becomes practically approachable.

In this paper, a unified Chebyshev surrogate model based hybrid uncertainty analysis framework is devised to robustly and efficiently assess uncertain nonlinear structural responses. A generalized unified interval stochastic sampling (GUISS) scheme is proposed, combining with the low-discrepancy sequences initialized high-order nonlinear particle swarm optimization (LHNPSO) algorithm. The probabilistic features of the lower and upper bounds can be adequately estimated for nonlinear random interval structural behaviours. Both random and interval uncertain parameters commonly encountered in engineering applications are accommodated in this analysis framework.

The paper is organized as follow. In Section 2, the governing formulations for solving stochastic interval geometrically nonlinear problems using finite element method are developed. In order to overcome the difficulties of such method when uncertainties are presented, the unified Chebyshev surrogate model based hybrid uncertainty analysis method is proposed in Section 3. Subsequently, two numerical examples are investigated by using the proposed approach, and compared with the dual sampling method in Section 4. Finally, some conclusions are drawn in Section 5.

2. Stochastic interval finite element analysis of structures with geometric nonlinearity

The typical approach for handling nonlinear problem is using incremental solution, in which the prescribed loads are discretised to a number of load steps [30]. By applying the finite element method on the principle of virtual work to the unknown state at time step $t+\Delta t$, it gives:

$$^{t+\Delta t}\mathbf{F} = {}^{t+\Delta t}\mathbf{R} \tag{1}$$

where $^{t+\Delta t}\mathbf{R}$ and $^{t+\Delta t}\mathbf{F}$ represent the vector of externally applied nodal point forces and the vector of nodal point forces equivalent to the internal element stresses at time $t+\Delta t$ respectively. If assuming the solution at time t is known, the internal nodal force at $t+\Delta t$ can also be approximated as follows:

$$^{t+\Delta t}\mathbf{F} = {}^{t}\mathbf{F} + {}^{t}\mathbf{K}\cdot\Delta\mathbf{U} \tag{2}$$

where $^{t}\mathbf{K}$ is the tangent stiffness matrix, which corresponds to the geometric and material properties at time t, $\Delta\mathbf{U}$ is the incremental displacement from time t to time $t+\Delta t$. Substituting Eq. (2) into Eq. (1) yields:

$$^{t}\mathbf{K}\cdot\Delta\mathbf{U} = {}^{t+\Delta t}\mathbf{R} - {}^{t}\mathbf{F} \tag{3}$$

and solving $\Delta\mathbf{U}$, the approximated displacement at the unknown state $t+\Delta t$ can be calculated as:

$$^{t+\Delta t}\mathbf{U} \doteq {}^{t}\mathbf{U} + \Delta\mathbf{U} \tag{4}$$

As Eq. (2) linearizes the configuration at time t, therefore Eqs. (3) and (4) only give an approximated displacement. In order to obtain more accurate solution, the same process needs to be applied in an iterative way, for $i = 1,2,3,\ldots$, as:

$$^{t}\mathbf{K}\cdot\Delta\mathbf{U}^{(i)} = {}^{t+\Delta t}\mathbf{R} - {}^{t+\Delta t}\mathbf{F}^{(i-1)} \tag{5}$$

$$^{t+\Delta t}\mathbf{U}^{(i)} = {}^{t+\Delta t}\mathbf{U}^{(i-1)} + \Delta\mathbf{U}^{(i)} \tag{6}$$

Subject to the initial conditions:

$$^{t+\Delta t}\mathbf{F}^{(0)} = {}^{t}\mathbf{F};\ {}^{t+\Delta t}\mathbf{U}^{(0)} = {}^{t}\mathbf{U} \tag{7}$$

Let $\zeta^R \in Z(\Re)$ where $Z(\Re)$ represent all possible random variables in a probability space$(\mathbf{\Omega},\mathbf{A},\mathbf{P})$, and $\Re$ denotes the set contains all real numbers. Also, $\xi^I = [\underline{\xi},\overline{\xi}] \Longleftrightarrow \{\xi \in \Re | \underline{\xi} \leqslant \xi \leqslant \overline{\xi}\}$ is an interval variable of $I(\Re)$ which denotes the set of all closed real intervals, where $\underline{\xi}$ and $\overline{\xi}$ denote respectively the lower and upper bound of ξ^I. When the uncertain system parameters are considered, the deterministic iterative nonlinear finite element analysis scheme can be reformulated into:

$$^{t}\mathbf{K}^{RI}\cdot\Delta\mathbf{U}^{RI^{(i)}} = {}^{t+\Delta t}\mathbf{R}^{RI} - {}^{t+\Delta t}\mathbf{F}^{RI^{(i-1)}} \tag{8}$$

$$^{t+\Delta t}\mathbf{U}^{RI^{(i)}} = {}^{t+\Delta t}\mathbf{U}^{RI^{(i-1)}} + \Delta\mathbf{U}^{RI^{(i)}} \tag{9}$$

with

$$^{t+\Delta t}\mathbf{F}^{RI^{(0)}} = {}^{t}\mathbf{F}^{RI};\ {}^{t+\Delta t}\mathbf{U}^{RI^{(0)}} = {}^{t}\mathbf{U}^{RI} \tag{10}$$

In Eqs. (8)–(10), $(\cdot)^{RI}$ represents the variable that is function of both random and interval uncertain parameters ζ^R and ξ^I such that:

$$\begin{cases} \zeta^R \in \mathbf{\Omega} := \{\zeta \in \Re^h | \zeta_d^R \sim g_{\zeta_d^R}(x), & for \quad d = 1,\ldots,h\} \\ \xi^I \in \mathbf{\Phi} := \{\xi \in \Re^v | \underline{\xi_c} \leqslant \xi_c \leqslant \overline{\xi_c}, & for \quad c = 1,\ldots,v\} \end{cases} \tag{11}$$

where ζ^R denotes the vector which collects all the h random variables presented in the system; $g_{\zeta_d^R}(x)$ denotes the probability density function (PDF) of the random variable ζ_d^R; $\mu_{\zeta_d^R}$ and $\sigma_{\zeta_d^R}$ represent the mean and standard deviation of random variable ζ_d^R; ξ^I denotes the interval vector which collects all the v interval variables presented in the system; $\underline{\xi_c}$ and $\overline{\xi_c}$ denote the lower and upper bound of the cth interval variable. In most of nonlinear analysis methods for structures or engineering systems, such iterative approach is popularly adopted within modern engineering applications. However, when hybrid uncertain parameters are considered, such time consuming incremental solution method becomes very difficult to implement.