Structural Safety 66 (2017) 1–16

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

Reliability-based topology optimization of uncertain building systems subject to stochastic excitation

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ARTICLE INFO

Article history:
Received 21 August 2015
Received in revised form 27 July 2016
Accepted 14 January 2017
Available online 2 February 2017

Keywords:
Reliability-based design
Topology optimization
Seismic engineering
Monte Carlo simulation
Stochastic systems

ABSTRACT

Topology optimization has traditionally been developed in a deterministic setting, notwithstanding the considerable uncertainties that generally affect both the system as well as the excitation. Therefore, the development of methods that are capable of describing the performance of uncertain systems in a fully probabilistic setting would represent an important step forward. In particular, the ability to consider reliability constraints written in terms of first excursion probabilities posed on systems driven by general stochastic excitation would allow a wide variety of important design scenarios to be modeled. This paper is focused on proposing a simulation-centered reliability-based topology optimization framework to this end. In particular, an approach is developed based on defining, from the argumentation of the simulation process, an optimization sub-problem that not only approximately decouples the probabilistic analysis from the optimization loop, but takes a form that can be extremely efficiently solved. By solving a limited sequence of sub-problems, solutions are found that rigorously meet the first excursion constraints of the original problem. A series of case studies are presented illustrating the potential of the proposed framework.

1. Introduction

Topology optimization is a powerful conceptual design tool that can be applied to a wide variety of engineering problems. The strength of topology optimization lies in its ability to systematically explore complex design spaces. Within the field of structural engineering, it can be used as a formal approach for the identification of optimal load-resisting systems [1–10]. Notwithstanding the considerable uncertainties that generally affect the system (e.g. dynamic and material properties) and excitation (e.g. earthquakes and wind) of many practical applications, the majority of work on topology optimization has been performed in a deterministic setting. This can lead to systems that exhibit poor performance, or even fail. In order to avoid this, the uncertainties affecting the problem must be rationally treated through, for example, reliability methods. In the case of uncertain systems with fixed topologies, reliability-based optimization methods are relatively well established (for a review see e.g. [11,12]). The same cannot be said for the topology optimization of uncertain system, where the difficulty in developing such a framework can be traced back to how topology optimization problems are inevitably characterized by large design variable vectors (in the order of thousands) that are necessary for adequately discretizing the design domain. When this is combined with systems driven by general stochastic excitation and subject to constraints on first excursion probabilities, a problem is defined that is not only in terms of a large design variable vector but also of high-dimensional implicit probabilistic integrals. This combination is challenging, as the presence of a large design variable vector significantly complicates the calculation of the sensitivities of the implicit reliability constraints, while making methods based on developing local or global approximations of the reliability constraints computationally intractable. Indeed, the majority of the research on reliability-based topology optimization (RBTO), or more generally topology optimization under uncertainty, has focused on static problems characterized by relatively few sources of uncertainty, as for example, material properties [13–16], applied static loads [13,14,17–21], nonstructural mass [18], or geometric uncertainties [16,17,19,22].

Recently, there has been growing interest in developing methods that can include stochastic excitation [7,8,10,23,24]. In particular, [23,24] have developed a method for deterministic structures subject to stochastic excitation modeled as a Gaussian process using a classic discrete representation method [25]. The approach is based on calculating the instantaneous failure probability, as

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http://dx.doi.org/10.1016/j.strusafe.2017.01.005
0167-4730/

opposed to the more general but challenging first excursion probability, through first order reliability methods (FORM). The approach is limited by the excitation model and the unavoidable limitations of FORM [26]. In Kareem et al. [8] and Bobby et al. [10] a framework is proposed for the topology optimization of uncertain systems subject to reliability constraints (in terms of fragility), where it was assumed that the excitation was stationary, Gaussian and ergodic, therefore leading to a closed form for the failure probability. This can be limiting, as the stochastic excitation generated by many natural hazards (e.g. earthquakes, but also thunderstorms, hurricanes, and tornadoes [27]) may be non-stationary and/or non-Gaussian. While it is traditional to assume stationarity in order to facilitate calculations, this inevitably introduces a source of model uncertainty during the determination of the system's reliability. The development of optimization frameworks that allow the use of non-stationary stochastic excitation would allow the user to decide the most appropriate reliability model to use for describing the performance of the system.

This paper is focused on the development of such a framework. In particular, a simulation-based RBTO framework is proposed for the conceptual design of uncertain structures subject to stochastic excitation and reliability constraints written in terms of first excursion probabilities.

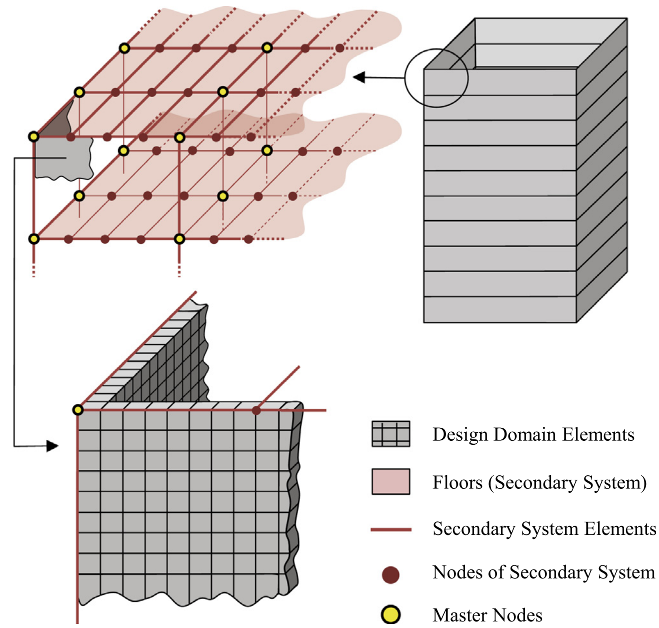

Fig. 1. Schematic of the discretized design domain and complete FE model.

2. Reliability-based topology optimization

2.1. Design domain

In order to apply topology optimization to the design of building systems, an appropriate design domain must first be defined. In particular, it is the lateral load-resisting system that is generally designed to withstand external environmental loads and is therefore of interest here. As a consequence, the design domain, i.e. the space in which elements of the lateral load-resisting system are permitted to exist, can be seen as a subset of the external envelope of the building. It is this domain, here indicated as Ω, that will be discretized with finite elements (FE) and then optimized. Because the generally non-designable secondary system (floor and gravity system) is an important and unavoidable source of stiffness for the lateral load-resisting system, it must also be modeled in order to ensure a realistic overall stiffness [10]. Therefore, an effective complete FE model of the system can be defined as the superposition of a FE model of the secondary system and a FE model discretizing the design domain, as shown in Fig. 1. If it is also observed that the majority of mass in a typical multistory building can be found at the floor levels [28], it becomes reasonable to consider the mass of the secondary system (including any carried mass) as well as the mass associated with the elements of the design domain concentrated at the floor levels. Because the flexibility of the floor systems must be considered, the mass cannot be lumped at a single node (e.g. the center of mass) per floor as is typical in building design [28]. Instead, the mass must be considered appropriately distributed over the floor area. From a modeling prospective, the distributed mass can be lumped at a number of master nodes belonging to the floor system as illustrated in Fig. 1.

If it is also ensured that the master nodes, i.e. mass nodes, belong to the FE model of the secondary system, for mass dependent loading (i.e. seismic or any other dynamic loading) the design domain becomes independent of the loading scheme. In other words, the external loads can always find a load path through the floor system to the main lateral load-resisting system. This is an extremely desirable property as it ensures that the resulting topologies are not guided by the choice of the master nodes. This will result in a greater optimality of the final designs while ensuring a realistic load path for mass dependent loads. In this work, the stiffness matrix of the complete system will be indicated as $\mathbf{K}$ while the corresponding mass matrix will be indicated as $\mathbf{M}$.

2.2. Problem setting

In order to find optimal topologies in the design domain identified above while considering an uncertain system subject to external stochastic excitation, solutions to the following reliability-based topology optimization must be found:

$$\begin{aligned} &\min_{\boldsymbol{\rho}} V(\boldsymbol{\rho}) = \sum_{e=1}^{n} \int_{\Omega_e} \rho_e \mathrm{d}\Omega \\ &\text{s.t.}\;\; \mathrm{P}\big(G_j(\boldsymbol{\rho}, \mathbf{U}_s, \mathbf{U}_g, \mathbf{X}) \leqslant 0\big) \leqslant \mathrm{P}_{0_j}, \quad j = 1, \ldots, N \\ &\qquad 0 \leqslant \rho_e \leqslant 1 \end{aligned} \tag{1}$$

where $\boldsymbol{\rho} = \{\rho_1, \ldots, \rho_n\}^T$ is the element-wise normalized material density design variable vector that is generally related to an independent design variable vector $\mathbf{y}$ through a local filter operator φ_e (e.g. the H-filter [29]) such that $\boldsymbol{\rho} = \varphi_e(\mathbf{y})$, n is the total number of elements composing the discretized design domain, Ω_e denotes the domain of element e, V is the volume of material in the design domain, G_j is the limit state function of interest for the jth reliability constraint while N is the total number of reliability constraints, P_{0_j} is the target failure probability for the jth reliability constraint, $\mathbf{U}_s$ is a vector of uncertain structural parameters, $\mathbf{U}_g$ is a vector of uncertain parameters modeling the intensity and epistemic uncertainties of the stochastic excitation, and $\mathbf{X}$ is a white noise sequence specifying the stochastic excitation. In the following, realizations of the vectors $\mathbf{U}_s$, $\mathbf{U}_g$ and $\mathbf{X}$ will be indicated in lowercase.

As mentioned in the introduction, the solution of the optimization problem defined in Eq. (1) is not trivial due to the considerable size of the design variable vector as well as the high-dimensional probabilistic integrals that define the reliability constraints. Indeed, the use of gradient-based algorithms is desirable for problems with large design variable vectors due to their efficiency; however, the implicit dependence of the reliability constraints on the design variables $\boldsymbol{\rho}$ significantly increases the computational effort needed to obtain their sensitivities. Although the use of approximate reliability methods (first and second order reliability