Structural Safety 66 (2017) 84–93

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

A hybrid relaxed first-order reliability method for efficient structural reliability analysis

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ARTICLE INFO

Article history:
Received 3 May 2016
Received in revised form 22 February 2017
Accepted 26 February 2017

Keywords:
Reliability analysis
First-order reliability method
Hybrid relaxed HL-RF
Angle criterion

ABSTRACT

The Hasofer-Lind and Rakwitz-Fiessler (HL-RF) algorithm is widely used for structural reliability analysis in first-order moment method (FORM). However, it meets non-convergence problem including generating periodic and chaotic solutions for highly nonlinear limit state function. In this paper, relaxed HL-RF (RHL-RF) is developed based on a relaxed factor, which is dynamically computed by the second-order interpolation between zero and one. A hybrid relaxed HL-RF (HRHL-RF) method is proposed, in which the HL-RF and RHL-RF are adaptively implemented by using an angle criterion to improve the robustness and efficiency of FORM formula. The angle condition is simply calculated based on the results from the new and previous points. Finally, the performances in terms of robustness and efficiency of the HRHL-RF are compared with several existing FORM methods through five mathematical and structural examples. The results indicate that HRHL-RF method is more robust than the HL-RF and more efficient than other existing methods.

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1. Introduction

Engineering problems inherently involve various uncertainties including material properties, external loads and geometrical dimensions. Therefore, the structural reliability methods provide a powerful tool to handle these uncertainties based on the limit state function (LSF) or performance function [1,2]. The main effort in structural reliability analysis is to approximate the probability of failure by solving the following integration [3]:

$$P[g(\boldsymbol{X}) \leqslant 0] = \int_{g(\boldsymbol{X})\leqslant 0} \cdots \int f_{\boldsymbol{X}}(\boldsymbol{X}) d\boldsymbol{X} \approx \Phi(-\beta) \quad (1)$$

where P_f is the structural failure probability, $g(\boldsymbol{X})$ is the LSF in X-space. When $g(\boldsymbol{X}) \leqslant 0$, it denotes the sample point $\boldsymbol{X}$ is located at failure region. $f_{\boldsymbol{X}}(\boldsymbol{X})$ is the joint probability density function for the random variables $\mathbf{X}$. Φ is the standard normal cumulative distribution function with respective to reliability index β. When high nonlinear limit state function with several random variables is involved, a general solution of above integration is difficultly obtained or is usually unavailable [3,1]. To address this issue, several reliability methods were developed to estimate the failure probability based on probabilistic model such as Monte Carlo simulation (MCS) [4], the first-order reliability method (FORM) [5–8] and second-order reliability method (SORM) [9]. The MCS is an accurate method but computationally unaffordable for real engineering problems. The FORM is a widely used approximate analytical method, because it makes a good balance between accuracy and efficiency in realistic engineering reliability analysis. In FORM, structural failure probability is estimated based on the reliability index (β) i.e. $P_f \approx \Phi(-\beta)$. $\boldsymbol{U}^*$ is the most probable point (MPP), which is the minimum distance point on the limit state surface ($g(\boldsymbol{U}^*) = 0$) to the origin in the standard normal space as [10,2]:

$$\begin{aligned} &\text{Find } \boldsymbol{U}^* \\ &\min\ (\boldsymbol{U}^T\boldsymbol{U}) \\ &S.t\ g(\boldsymbol{U}) = 0 \end{aligned} \quad (2)$$

The Hasofer-Lind and Rakwitz-Fiessler method (HL-RF) formula are commonly applied to evaluate the probabilistic model using FORM, but it may provide unstable results including periodic and chaotic solutions for highly nonlinear problems [11–14]. To improve the stability of the HL-RF method, various attempts can

Abbreviations: MCS, Monte Carlo simulation; FORM, first-order reliability method; SORM, second-order reliability method; LSF, limit state function; HL-RF, Hasofer-Lind and Rakwitz-Fiessler; FSL, finite-step length; IHL-RF, improved HL-RF; CC, chaos control method; CHL-RF, conjugate HL-RF; RHL-RF, relaxed HL-RF method; HRHL-RF, hybrid relaxed HL-RF method; MPP, most probable point.

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http://dx.doi.org/10.1016/j.strusafe.2017.02.005
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Nomenclature

$\boldsymbol{X}$	random variable vector	$\nabla g(\cdot)$	gradient vector of performance function
$\boldsymbol{U}$	normalized random variable vector	$[\boldsymbol{J}]$	Jacobian matrix
$\boldsymbol{U}^*$	most probable point	ζ	chaos control factor or finite-step size
$\boldsymbol{d}$	search direction vector	P	peak factor
β	reliability index	P_f	failure probability
$\tilde{n}_k^{\zeta}$	finite redial direction vector	$\Phi(.)$	cumulative distribution function
d_k^{ζ}	conjugate search direction	θ	angle between search direction vectors
ε	stopping criterion	$f_X(\boldsymbol{X})$	joint probability density function
ε_θ	small value to satisfy angle criterion	λ	penalty coefficient
α	relaxed factor or step size	B	representation of model uncertainty
μ	mean vector	$\rho(\boldsymbol{J})$	spectral radius of Jacobian matrix
σ	standard deviation vector	$\tilde{n}_k^{c\zeta}$	conjugate radial direction vector
$g(\boldsymbol{X})$	limit state function in X-space	$\tilde{n}_k$	radial directional vector

be found in the literatures. Liu and Der Kiureghian [15] proposed the improved HL-RF (IHL-RF) through a merit function to overcome the non-convergence problem of the HL-RF method. Wang et al. [16] improved the HL-RF method based on the intervening variables and used the adaptive nonlinear two-point approximation of the limit state function. Santosh et al. [17] employed the Armijo rules to improve the HL-RF method. Yang [12] suggested the chaos control (CC) method to improve robustness of FORM. Gong and Yi [18] constructed a finite-step length (FSL) algorithm for MPP search.

Recently, Meng et al. [19,20] proposed modified CC (MCC) method based on a directional steepest descent search direction to improve the efficiency of CC method. The MCC is more efficient than the CC but is computationally inefficient for convex performance functions [21]. In the study of Keshtegar [22], the conjugate search direction with chaotic control factor was constructed to improve the robustness and efficiency of FORM. Keshtegar [8,14,23] further adopted sufficient descent criterion and Armijo rule to enhance the robustness of the conjugate FORM formulas. Generally, the improved versions of FORM, such as IHL-RF, CC, FSL and MCC methods, requires more number of function calls than the HL-RF for moderate nonlinear performance functions, because the step size/control factor is less than 1 among them. Therefore, the main challenge of a stable FORM formula is to obtain the MPP with less computational demand. The step size/control factor is the key for these improved FORM formula, which determines the stable convergence and efficiency of the reliability analysis process.

In this paper, a novel algorithm of FORM is proposed to improve the efficiency and robustness of MPP search method based on relaxed approach. The proposed relaxed method is established based on the results of the HL-RF at new iteration and the relaxed approach at previous iteration. A relaxed factor is formulated by using second-order interpolation to enhance the robustness of relaxed HL-RF (RHL-RF). To ensure the efficiency of RHL-RF, a hybrid iterative algorithm is presented by combing the merits of HL-RF and RHL-RF with an angle condition. The proposed hybrid relaxed HL-RF (HRHL-RF) is compared with the HL-RF, IHL-RF, CC, FSL, conjugate HL-RF (CHL-RF) and MCC methods by several mathematical and structural reliability examples.

2. The HL-RF iterative algorithm

Hasofer and Lind [5] method is well-known in the structural reliability analysis. Rackwitz and Fiessler [6] extended the Hasofer and Lind algorithm to approximate the failure probability. For brevity, this algorithm is denoted herein as the HL-RF. The iterative HL-RF scheme for searching the MPP is given as [11]

$$\boldsymbol{U}_{k+1}^{HL} = \frac{\nabla^T g(\boldsymbol{U}_k)\boldsymbol{U}_k - g(\boldsymbol{U}_k)}{\nabla^T g(\boldsymbol{U}_k)\nabla g(\boldsymbol{U}_k)}\nabla g(\boldsymbol{U}_k) \tag{3}$$

where $\nabla g(\boldsymbol{U}) = [\partial g/\partial U_1, \partial g/\partial U_2, \ldots, \partial g/\partial U_n]^T$ is the gradient vector of the limit state function at the point $\boldsymbol{U}$. The iterative HL-RF in Eq. (3) shows the MPP search is dependent on the gradient vector of the limit state function at $\boldsymbol{U}$. This means that the steepest descent search direction is implemented in HL-RF, thus the nonlinearity degree of limit state in normal standard space is an important factor for stabilizing the HL-RF formula. $\boldsymbol{U}$ is the standard normal standard variable, which is calculated as follows [8]:

$$\boldsymbol{U} = \frac{\boldsymbol{X} - \boldsymbol{\mu}_X^e}{\boldsymbol{\sigma}_X^e} \tag{4}$$

where, μ_X^e and σ_X^e are the equivalent means and the standard deviations of the basic random variables **X**, respectively. The equivalent means and standard deviations of non-normal random variables are computed based on the Rosenblatt transformation i.e. $\boldsymbol{U} = \Phi^{-1}[F_{\boldsymbol{X}}(\boldsymbol{X})]$ as follows [3]:

$$\boldsymbol{\sigma}_X^e = \frac{1}{f_{\boldsymbol{X}}(\boldsymbol{X})}\varphi[\Phi^{-1}\{F_{\boldsymbol{X}}(\boldsymbol{X})\}] \tag{5}$$

$$\boldsymbol{\mu}_X^e = \boldsymbol{X} - \boldsymbol{\sigma}_X^e\Phi^{-1}[F_{\boldsymbol{X}}(\boldsymbol{X})] \tag{6}$$

where $F_{\boldsymbol{X}}(\cdot)$ and $\Phi^{-1}(\cdot)$ are the cumulative distribution function and inverse standard normal cumulative distribution function, respectively. According to Eqs. (3)–(6), the MPP search is depended on statistical characteristics of random variables, so all these random variables should be transformed into standard normal variables $\boldsymbol{U}$ as

$$\boldsymbol{U} = f(\boldsymbol{X}, \boldsymbol{\mu}_X^e, \boldsymbol{\sigma}_X^e) \tag{7}$$

Based on Eq. (7), the convergence of HL-RF scheme is related to the means and standard deviations of basic random variables.

3. Improved FORM algorithms

3.1. Imroved HL-RF iterative method

The improved HL-RF (IHL-RF) is one popular FORM iterative algorithm which is formulated as follows [15]:

$$\boldsymbol{U}_{k+1}^{IHL} = \boldsymbol{U}_k^{IHL} + \alpha\boldsymbol{d}_k^{IHL} \tag{8}$$

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