Structures 12 (2017) 179–187

Contents lists available at ScienceDirect

Structures

journal homepage: www.elsevier.com/locate/structures

Geometric Nonlinear Analysis of Plane Frames With Generically Nonuniform Shear-deformable Members

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ARTICLE INFO

Keywords:
Direct Stiffness Method (DSM)
Plane frame elements with axially varying rigidity
Geometric nonlinear analysis
Cross-sections of arbitrary shapes

ABSTRACT

This paper employs the Direct Stiffness Method (DSM) to carry out geometric nonlinear analysis of plane frames with nonuniform physical-geometric characteristics. At the element level, a flexibility system of equations based on the principle of virtual forces (PVF) is established to calculate the tangent stiffness matrix and the equivalent nodal loads. The formulation allows for the easy modeling of shear-deformable frame elements with generic rigidity variation along their axes. In addition, Green's theorem is considered to express all the necessary section properties in terms of boundary integrals. This considerably simplifies the modeling of complex cross sections of arbitrary shapes. A "boundary-element" mesh is then used to model the geometric description of the cross sections. At the structure level, to determine the nonlinear equilibrium trajectories for the frame, we apply a co-rotational updated Lagrangian formulation along with an incremental-iterative full Newton-Raphson process. Large displacements and internal member forces are accurately reconstituted. Frameworks having elements with geometrically complex cross-sections varying along their axes are analyzed to validate the strategy proposed.

1. Introduction

In engineering practice, engineers must possess computational analysis tools that have the ability to model frame structures containing elements with cross sections of arbitrary shapes, and/or variable rigidities. To reduce material consumption, for instance, engineers may vary the sectional dimensions of steel or reinforced concrete members along their axes. In concrete structures, the rigidity is essentially variable because of cracking [1]. Thus, designing real frame structures generally calls for strategies that take into account the nonuniform physical and geometric characteristics of the elements.

In this paper, to conduct a nonlinear analysis of 2D frame structures with these complex physical-geometric characteristics, the Direct Stiffness Method (DSM) is employed. DSM may be regarded as the starting point from which the Finite Element Method (FEM) evolved. In fact, it offers the simplest way to implement FEM, taking into account 2-node elements. In this respect, to quote one of the founders of FEM, Ray W. Clough, FEM may be viewed as a mere "*extension of standard methods of structural analysis in which the structure is treated as an assemblage of discrete structural elements*" [2].

In standard finite element (FE) formulations for frame structures [3], interpolation functions for deflections must be known in advance to allow for computing the stiffness matrices and corresponding equivalent load vectors. If FE formulations are to be developed then in this way, the fact that it is impossible to obtain exact interpolation functions in real-life situations – say, shear-deformable frame elements under generic rigidity variation – makes it also impossible to determine their exact stiffness matrices and corresponding equivalent nodal load vectors [4]. One way out of this conundrum is to approximate the tapered element via a number of uniform elements for which the exact displacement functions are known [4,5].

Researchers have proposed more accurate solutions for this class of problems by tackling the differential equation of equilibrium for beams, which have variable coefficients. Coming up with closed-form solutions is possible, however, only in very simple cases, usually for linearly tapered elements and not including shear-deformation effects [4,6]. In [7], the authors employed the concept of transfer functions based on Bernoulli's beam theory to develop a 3D-beam, 2-node, finite element that permitted the modeling of continuous cross-section variations along the element axis. In that procedure, an n-order polynomial is considered to describe the variation of the sectional properties. In principle, this strategy may be used to model general cross-sectional shapes. For its validation, however, the authors in [7] considered only common ones (rectangular and circular). Furthermore, that element includes no shear deformation.

In [8], Chebyshev polynomials are used to obtain the stiffness matrices for tapered plane frame elements based on Timoshenko-Euler's beam theory, in which both shear and axial-force deformations are

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http://dx.doi.org/10.1016/j.istruc.2017.09.002
Received 7 July 2017; Accepted 11 September 2017
Available online 19 September 2017
2352-0124/

simultaneously considered. The accuracy of this strategy, though, is dependent on the number of terms taken into account in the Chebyshev polynomials used to represent the deflection as well as the rigidity-based parameters involved. Yet this strategy might be inefficient for generic rigidity variations. Moreover, no criterion is stated for determining a reasonable number of terms in the Chebyshev polynomials.

Rather than look for a solution to the differential equations governing the equilibrium in beams, Friedman and Kosmatka [9,10], applied the Castigliano's second theorem to derive a flexibility-based method that allows for writing the analytical expressions of the stiffness coefficients in Bernoulli-Euler's and Timoshenko's beams with generic rigidity variations. Proceeding in such a manner avoids the drawback of standard FE formulations concerned with the prior knowledge of displacement interpolation functions. Recently, de Araújo and Pereira [11] proposed a flexibility method based on the principle of virtual forces (PVF) to obtain closed expressions for the stiffness coefficients and the consistent equivalent load values for 3D (space) shear-deformable frames under generic variation of rigidity. In PVF, the internal virtual work is due to the action of virtual generalized stresses on real generalized strains. By writing the latter ones in terms of real generalized stresses (real internal forces), this procedure enables then the easy representation of the exact stiffness values and consistent equivalent nodal loads in complex problems.

Of course, getting these exact values requires the exact evaluation of the available strain energy products over the element length, which will be essentially dependent on the representation of the internal forces, and on the rigidity variation along the element. To amplify the modeling resources of the computational technique, this work adopts suitable polynomial interpolations to approximate the rigidity values and internal forces at the element. To evaluate the available integrals, this work employs standard Gauss–Legendre quadratures. In [11], this strategy is applied to construct elastic stiffness matrices and equivalent nodal load vectors for generic 3D (space) frame elements; these include flexural, axial, shear, and torsional deformation modes. In fact, the proposed procedure can be applied to accurately calculate all consistent structural matrices (including mass and damping ones), and consistent load vectors for any frame element, under the most generic physical-geometric cases. In the current paper, this technique is extended to determine the geometric stiffness matrices for generic 2D (plane) shear-deformable frame elements.

In Timoshenko's beam theory, an important and somewhat controversial issue is the consideration of shearing stress/strain effects. Regarding this consideration, the literature offers several definitions for the so-called shear correction factor [12–18]. In general, one aims to replace the actual shearing stress distribution in a certain direction, on a cross-section, with an equivalent constant one on an effective shear area. Thus, depending on the criterion one adopts to find this equivalent constant stress distribution, one may obtain somewhat different shear correction factors. To determine them with rigor, researchers have considered 3D elasticity solutions. Results reported in the literature [15,17,19] have shown these solutions' dependence on Poisson's ratio and even on the aspect ratio of the cross-section [17].

On the other hand, Freund and Karakoç [20] recently showed, based on a refined Timoshenko's beam model, that shear correction factors are indeed purely geometric parameters, depending exclusively on the cross-sectional shape. Regardless, even if determined as an elastic section parameter, the variation of shear correction factors as a function of Poisson's ratio is not significant at all [19]. In this study, we simply adopt a shear-correction factor based on solutions from elementary beam theory. The factor is determined by equaling the shear-strain energy associated with the actual shearing stresses on a given cross-section to that of an equivalent constant shearing stress on a corresponding effective shear area. This shear-correction factor corresponds exactly to the one adopted by the commercial code SAP2000 [21].

It should be noted that to compute all needed section properties as areas, the first and second moments of area as well as the shear-stress form factors (the inverses of the shear-correction factors), all of the domain integrals involved are converted into boundary integrals, and "boundary-element" meshes are used to numerically calculate them. Doing so makes especially efficient the computational modules for determining all the element cross-section properties. This paper describes in detail the algorithm for evaluating shear-stress form factors.

Finally, to verify the proposed procedure's suitability and correctness for calculating stiffness matrices and equivalent load vectors, we incorporated it into a code for the non-linear analysis of plane frames. In the non-linear analysis, we employ an updated co-rotational formulation along with a standard full Newton-Raphson incremental-iterative method. The results obtained for steel structures with complex cross-sectional geometries are compared with those obtained by other authors and using the SAP2000 [21].

2. The DSM coefficients for plane frame elements

This paper presents a numerical process for calculating the DSM stiffness matrices (including the geometric one) and the equivalent nodal load vectors for plane frame elements with cross sections of any geometric shape, and generically varying along their centroidal axis. For this class of structures (plane frames), the structural finite element is that shown in Fig. 1, with 6 degrees of freedom. The stiffness coefficients k_{ij} correspond to the element actions for unit prescribed displacements $u_j = \delta_{ij}$, $j = 1,2,\ldots,6$, in the direction of the degrees of freedom, while the equivalent nodal loads f_{i0}, $i = 1,2,\ldots,6$, associated

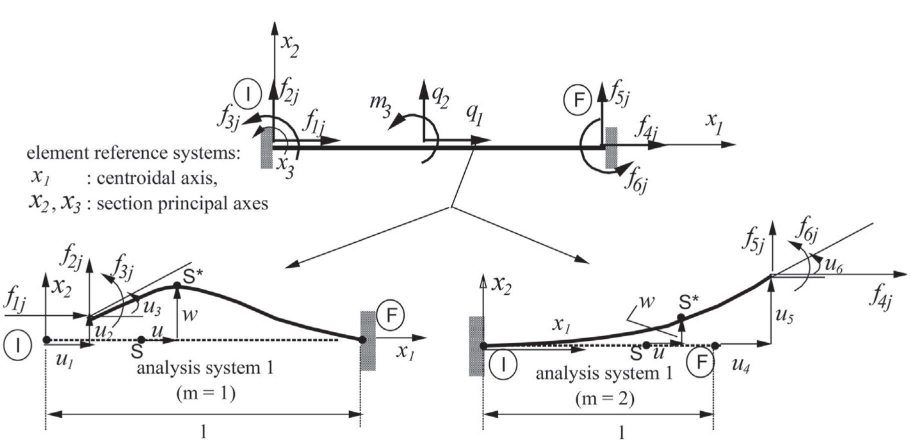

Fig. 1. 2D frame element at the undeformed and deformed configurations.