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Mathematical Model to Determine the Weld Resistance Factor of Asymmetrical Strength Results

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ABSTRACT

A normal distribution model is usually used to determine the structural steel resistance factors. This distribution has a symmetric probability density curve. When test results are skewed, a normal distribution model is not an efficient mathematical model to determine the resistance factors. Other mathematical models which have a skewed density probability curve should be used in such situations. The objective of this study is therefore to determine the mathematical model that is most suitable to determine the resistance factors of skewed set of test results. To accomplish this, 21 weld strength values were used in this investigation to determine the weld resistance factor. A histogram of the weld strengths illustrated that the weld strengths are skewed. Distributions are skewed, especially if the values under investigation cannot be negative, the mean are low and the variances are large. Since weld strengths are skewed, a beta distribution, chi-square distribution and gamma distribution were used to analyse the results, and the mathematical function, which was found to fit the sample data best was selected and used to compute the design strength of the 21 weld strengths, using 0.01 probability of failure. Among the distribution analysed, the beta probability distribution was found to be the most effective model.

1. Introduction

The responsibility of a structural engineer is to design structures that are safe from failure or malfunction during their design lifetime. To ensure that structures are safe, design rules, which enables that the strengths or resistances of the structure are greater than the loads that the structure experience have been established and packaged as standards. In South Africa, such standards include SANS 10162-1 [1], SANS 10162-2 [2] and SANS 10162-4 [3]. SANS 10162-1 [1] covers the limit-states design of hot-rolled steelwork, SANS 10162-2 [2] covers the limit-states design of cold-formed steelwork and SAN S10162-4 [3] covers the design of cold-formed stainless steel structural members. Whilst SAN S10162-1 [1] is based on the Canadian steel standard, CAN/CSA-S16-09 [4], SANS 10162-2 [2] is based on the Australian/New Zealand standard, AS/NZS 4600 2005 [5] and SANS 10162-4 [3] is based on the SEI/ASCE 8-02 standard [6]. In these standards, safety is achieved by multiplying the load effects with load factors that are greater than one, and thus increase the capacity of the load, while the strengths or resistances of the structures are multiplied by a resistance factor of less than one to reduce the capacity of the structure or structural elements. However, structures are not completely free from failure, since load effects and the strengths are random variables that cannot be explicitly determined at the design stage. Thus, there is always a small probability of failure for every structure that is designed to give a structure an acceptable level of safety. In South Africa, the probability of failure of steel structures or elements is limited to 1%. Since all the uncertainties are not known during the design process, probability theory is used to determine safety factors that will account for the uncertainties [7]. These safety factors are then employed in the design equations to make sure that the structures designed perform satisfactory.

According to SANS 10162-1, the factored shear strengths of the weld is defined as given in Eq. (1).

$$V_r = 0.67\varphi_w A_w f_{uw}(1.00 + 0.5\sin^{1.5}\theta) \quad (1)$$

where, φ_w is the resistance factor, A_w is the area of the effective weld throat, f_{uw} is the tensile strength of the weld metal and θ is the angle of loading measured from the weld longitudinal axis, in degrees. For longitudinal loading, $\theta = 0$, the parenthetical term in the above equation becomes 1, yielding the same allowable unit stress as has been traditionally permitted in previous standards. For perpendicular loading, $\theta = 90^{\circ}$, the parenthetical term becomes 1.5, allowing for the increase in the unit stress. Eq. (1) was derived from extensive studies by Butler and Kulak [8], Miazga and Kennedy [9], Lesik and Kennedy [10], Ng et al. [11], Iwankiw [12] and Kanvinde et al. [13] on fillet welds. The resistance factor of fillet welds in this equation is $\phi_w = 0.67$. This factor takes into account the variability of material properties, dimensions, workmanship, type of failure and uncertainty in the prediction of

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connection resistance. The limitation with this resistance factor is that it was derived, based on a normal probability distribution. The basis of this model is that when the sample size is large, the distribution of random variables will be normal or nearly normal. A log-normal distribution is used exclusively in structural steel to determine the reliability index, as the probability of a normal distribution is not zero for negative values. Sometimes, as demonstrated by Fraczek et al. [14], the normal distribution and the log-normal distribution are not sufficient to model strengths results. This normally happens when the strengths results are extremely skewed. In a previous study, Dundu and Krige [15] and Dundu [16,17] assumed a normal distribution, when some of the weld strength results analysed were skewed due to the use of different levels of welders.

A very important part of this investigation is to select a model that can accurately describe the test results. In this paper, the selection of the probability distribution is guided by a histogram, where discrete or continuous data is divided into classes or groups, often referred to as bins. Once the model has been selected, the model parameters can be estimated using several methods, and hence fit the model to the data. Various techniques are available for fitting data to models, and these include, a cumulative frequency, fit by areas and the method of moments. Since the weld strengths in this study are skewed, a beta distribution, chi-square distribution and a gamma distribution were used to find the probability of failure of 1% of the 21 weld strengths. Finally, the strength of the welds, equivalent to a probability of failure of 1%, is used to determine the weld resistance factor. A resistance factor account for the uncertainties such as material properties, construction practices and the precision of analysis, long term material performance, accuracy of the design theory and natural variation of loads used in the design process [18]. The philosophy used to determine the weld resistance factor is similar to the philosophy adopted by Dundu [16,17].

2. Probability distribution models

Knowledge of available probability distribution is extremely useful in establishing the rational and effectiveness of the method used to evaluate the data. A brief review of the probability distributions investigated in this study is given below.

2.1. Normal probability distribution

The normal probability distribution is the most widely used probability distribution in statistics [19]. This is perhaps due to the fact that the normal distribution is easy to apply in comparison to other statistical distributions. The other reason in favour of this model is that when the number of random variables is sufficiently large they assume a normal distribution. The limitations of the normal probability distribution are its symmetry (zero skewness) and its inability to make the probability density of negative values zero. Furthermore, it is defined mainly by the mean and standard deviation. Changes in the value of the mean translate the midpoint, but do not alter the shape of the curve, however changes in the standard deviation, greatly affect its shape. The probability density function (PDF) of a normal probability distribution is given by Eq. (2).

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]; \quad -\infty < x < \infty \tag{2}$$

In Eq. (2), μ is the mean or median or location parameter, σ is the standard deviation and σ^2 is the variance. Eq. (2) shows that the probability would not be zero when the values of the sample are negative. When $\mu = 0$ and $\sigma = 1$, the normal probability distribution is referred to as the standard normal distribution. The normal density probability distribution has a symmetric curve because the location parameter is equal to the mean of the values. The normal cumulative curve is determined using Eq. (3).

$$f(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \tag{3}$$

2.2. Log-normal probability distribution

To overcome some of the limitations of a normal probability distribution, a log-normal probability distribution is normally used. In a log-normal probability distribution, random variables are simply transformed into natural logarithms of random variables, which are normally distributed [19]. A major difference with the normal probability distribution is in the shape of the distribution. Where the normal distribution is symmetrical, a lognormal is skewed to the right or positively skewed, since all the values in a log-normal distribution are positive. Another distinction is in the assumption that the values used to derive a log-normal distribution are normally distributed. The density and the cumulative distribution function of a log-normal distribution is described by Eqs. (4) and (5), respectively.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma}\right)^2\right]; \quad 0 < x < \infty \tag{4}$$

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma}\right)^2\right] dx \tag{5}$$

2.3. Gamma distribution

A gamma distribution is another probability distribution model that can be used to model skew results, and takes its name from the well-known gamma function that has been studied in many areas of mathematics [20]. An advantage of the gamma distribution is its ability to represent a range of distributions, utilising only two parameters (the scale and shape). However, just like the log-normal distribution model, the gamma distribution is positively skewed. The probability density function of the gamma distribution is expressed as shown in Eq. (6).

$$f(x) = \frac{\left(\frac{x-\mu}{\beta}\right)^{\alpha-1} \exp\left(-\frac{x-\mu}{\beta}\right)}{\beta\Gamma(\alpha)}; \quad 0 < \beta, \alpha; \mu \le x \tag{6}$$

where, μ is the location parameter, α is the shape parameter, β is the scale parameter, and Γ is the gamma function. The gamma function is expressed by

$$\Gamma(\alpha) = \int_{0}^{\infty} t^{\alpha-1} e^{-t} dt \tag{7}$$

and the cumulative distribution function of the gamma distribution is given by

$$F(x) = \int_{0}^{x} \frac{\left(\frac{x-\mu}{\beta}\right)^{\alpha-1} \exp\left(-\frac{x-\mu}{\beta}\right)}{\beta\Gamma(\alpha)} = \frac{\Gamma_x(\alpha)}{\Gamma(\alpha)} \tag{8}$$

2.4. Chi-square distribution

A distribution of the sum of the squares of independent standard random variables is referred to as the Chi Square (χ^2) distribution [21]. It is a particular case of the gamma distribution in which the shape parameter α is n/2, location parameter μ is 0 and scale parameter β is 2. It possesses positive skewness, which approximate the normal distribution model as the degree of freedom increases. The "n" in the shape parameter refers to the positive integer, often called the chi-square