Structures 10 (2017) 30–38

Contents lists available at ScienceDirect

Structures

journal homepage: www.elsevier.com/locate/structures

Full Plastic Resistance of Tubes Under Bending and Axial Force: Exact Treatment and Approximations

J. Michael Rotter, Adam J. Sadowski *

Department of Civil & Environmental Engineering, Imperial College London, UK

ARTICLE INFO

Article history:
Received 9 September 2016
Received in revised form 15 November 2016
Accepted 22 November 2016
Available online 25 November 2016

Keywords:
Tubular cross sections
Cylindrical shells
Full plasticity
Bending moment – axial force interaction
Approximations
Design

ABSTRACT

The full plastic resistance under a combination of bending and axial force of tubes of all possible wall thicknesses, from thin cylinders to circular solid sections, does not ever seem to have been thoroughly studied, despite the fact that this is a relatively simple analysis. The first part of this paper presents a formal analysis of the state of full plasticity under longitudinal stresses in a right circular tube of any thickness free of cross-section distortions. The derivation leads to relatively complicated algebraic expressions which are unsuitable for design guides and standards, so the chief purpose of this paper is to devise suitably accurate but simple empirical descriptions that give quite precise values for the state of full plasticity whilst avoiding the complexity of a formal exact analysis. The accuracy of each approximation is demonstrated. The two limiting cases of a thin tube (cylindrical shell) and circular solid section are shown to be simple special cases.

The approximate expressions are particularly useful for the definition of the full plastic condition in tension members subject to small bending actions, but also applicable to all structural members and steel building structures standards, as well as to standards on thin shells where they provide the full plastic reference resistance. These expressions are also useful because they give simple definitions of the orientation of the plastic strain vector, which can assist in the development of analyses of the plastic collapse of arches and axially restrained members under bending.

1. Introduction

The full plastic resistance of tubes of all possible wall thicknesses and under all combinations of bending and axial forces does not ever seem to have been thoroughly studied, despite the fact that this is a well-defined problem that requires only a fairly simple analysis. However, the derivation leads to relatively complicated algebraic expressions which are unsuitable for design guides and standards, so the main purpose of this paper is to devise suitably accurate but simple descriptions that give quite precise values for the state of full plasticity whilst avoiding the complexity of a formal exact analysis. Because the condition of full plasticity of the perfect undeformed structure using ideal elastic-plastic material properties is one of the key reference states used in design rules [8,9,16,17], it is important that this state should be accurately defined.

It seems very likely that others may have performed the formal exact analysis for the full plastic condition under both bending and axial force long ago, but the authors have only traced the work of [21] after the review of this paper. Written in French and in a special revue, it was somewhat inaccessible. There consequently seems to be no identifiable basis for the rather varied full plastic interaction expressions used in current standards (e.g. [1–3,8,15]). The focus of this paper is on the development of suitable approximations for application in design guides and standards, as some of the existing approximate rules in standards are shown to be surprisingly inaccurate for such a formally precisely-defined problem.

The formal algebraic analysis of the state of full plasticity in a tube of any thickness is presented here, with the two limiting cases of a thin tube (cylindrical shell) and circular solid section shown as special cases of the full relationship. Because the general equations are too complicated for use in design calculations, two different sets of approximate formulas are presented together with a demonstration of the level of approximation associated with each. Simpler approximations produce greater errors.

These expressions are useful for the definition of the full plastic condition in tubular structural members, with special application for tension members subject to small bending actions, but also applicable to steel building structures standards and standards on thin shells where they provide the full plastic small displacement theory reference resistance. These expressions are also useful because they give simple definitions of the orientation of the plastic strain vector, which can assist in the development of plastic analyses of a particular class of redundant

* Corresponding author.
E-mail address: a.sadowski@imperial.ac.uk (A.J. Sadowski).

http://dx.doi.org/10.1016/j.istruc.2016.11.004

structures, including arches and axially restrained members under bending.

It should be recognised that the ultimate resistance of tubular members is affected by many other phenomena: elastic and elastic-plastic stability [7,11,19], ovalisation in long members under bending [4–6, 12,13,18], nonlinearity of the stress-strain relationship in metals other than mild steel [14] and geometric imperfections [10]. All these effects modify the resistance significantly, but the reference resistance against which all these modifications are made is the fully plastic state using an ideal elastic-plastic constitutive law and the undeformed perfect geometry [20]. For this reason, the analysis presented here gives the basic reference case, and it is important that it should be defined with precision.

2. Full plastic cross-section analysis

2.1. Introduction

In line with the terminology used in the Eurocode standard [8], the geometry of a tubular cross-section is here characterised by an external diameter d and a wall thickness t, as shown in Fig. 1. The analysis here treats the material as ideally plastic, with a simple linear yield boundary between the tension and compression zones. The yield boundary is deemed to satisfy the condition of plane sections remaining plane, leading to a straight linear boundary. Because the circular tube is symmetrical about its longitudinal axis, all orientations are identical and only a single orientation needs to be considered for conditions that might be regarded as biaxial bending in a different axis system.

2.2. Reference full plastic resistances under the action of individual stress resultants

The full plastic axial force N_{pl} for a circular tubular cross-section of external diameter d, thickness t and yield stress f_y is simply given by:

$$N_{pl} = \frac{1}{4}\pi\left(d^2-(d-2t)^2\right)f_y = \pi dt\left(1-\frac{t}{d}\right)f_y \tag{1}$$

For the limiting case of a circular solid rod ($d/t \to 2$ or $t/d \to 1/2$), this simplifies to:

$$N_{pl} = \frac{\pi}{4}d^2 f_y \tag{2}$$

For the limiting case of a thin tube ($d/t \to \infty$ or $t/d \to 0$), the (t/d) term becomes negligible and Eq. (1) simplifies to:

$$N_{pl} = \pi dt f_y \tag{3}$$

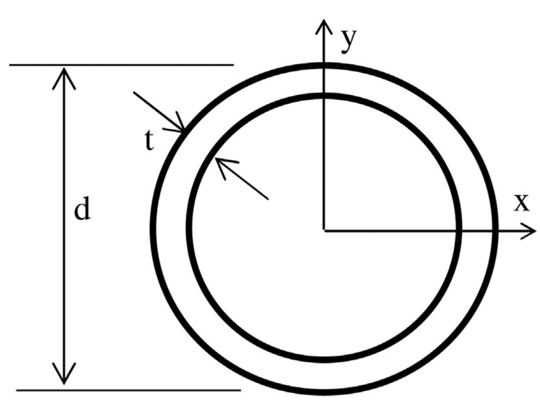

Fig. 1. Dimensions of the cross-section.

Similarly, the full plastic moment M_{pl} for a finite-thickness tube is given by:

$$M_{pl} = \frac{1}{6}\left(d^3-(d-2t)^3\right)f_y = \frac{4}{3}d^3\left(\frac{3}{4}\left(\frac{t}{d}\right)-\frac{3}{2}\left(\frac{t}{d}\right)^2+\left(\frac{t}{d}\right)^3\right)f_y \tag{4}$$

For a solid rod, the limiting case is:

$$M_{pl} = \frac{1}{6}d^3 f_y \tag{5}$$

For a thin tube, the terms $(t/d)^2$ and $(t/d)^3$ become negligible as $t/d \to 0$ and the limiting case is:

$$M_{pl} = d^2 t f_y \tag{6}$$

2.3. Reduced plastic moment in the presence of axial force for the two limiting cases of a solid rod and thin tube

It is appropriate to present briefly the interaction relationship of the plastic moment capacity under the effect of an axial force for the two simpler cases of a solid rod ($d/t \to 2$ or $t/d \to 1/2$) and a thin tube ($d/t \to \infty$ or $t/d \to 0$), as these form the two limiting reference cases against which the more complex relationship of the finite-thickness tube may be verified.

A fully-plastified circular cross-section under a moment M about the centroid and axial force N acting through the centroid undergoes yielding in different proportions in tension and compression depending on the relative magnitudes of these stress resultants (Fig. 2). The Yield Boundary (YB) between tension and compression intersects the exterior surface of the tube at an angle α from the vertical. For the case of pure bending ($N = 0$), $\alpha = \pi/2$ and the YB is coincident with the Centroidal Axis (CA) parallel to the YB.

Due to the doubly-symmetric nature of circular geometries, only the interaction between an axial force in one sense (either tension or compression) ($0 \le \alpha < \pi/2$) and a moment acting in one sense (either sagging or hogging) needs to be considered to obtain the full relationship. For clarity, the image in Fig. 2 shows a section with a larger zone in compression and a smaller zone in tension, but this choice is arbitrary. With this state of plasticity, the yield boundary YB in Fig. 2 moves from lying through the centroid and partitions the lower half of the cross-section into areas under tension A_{T1} and compression A_{C1}. These areas support net forces F_{C1} and F_{T1} acting through the respective centroids of those areas located at distances of y_{C1} and y_{T1} respectively from the CA. The area components and their centroidal distances from the CA may be determined from elementary geometry.

For a solid rod, these are:

$$A_{T1} = \frac{1}{8}d^2(2\alpha-\sin2\alpha) \quad y_{T1} = \frac{2}{3}d\left(\frac{\sin^3\alpha}{2\alpha-\sin2\alpha}\right) \tag{7a, b}$$

$$A_{C1} = \frac{1}{8}d^2(\pi-(2\alpha-\sin2\alpha)) \quad y_{C1} = \frac{2}{3}d\left(\frac{1-\sin^3\alpha}{\pi-(2\alpha-\sin2\alpha)}\right) \tag{7c, d}$$

$$A_{C2} = \frac{1}{8}\pi d^2 \quad y_{C2} = \frac{2}{3}\frac{d}{\pi} \tag{7e, f}$$

For a thin tube, these are:

$$A_{T1} = \alpha dt \quad y_{T1} = \frac{1}{2}d\frac{\sin\alpha}{\alpha} \tag{8a, b}$$

$$A_{C1} = \left(\frac{\pi}{2}-\alpha\right)dt \quad y_{C1} = d\left(\frac{1-\sin\alpha}{\pi-2\alpha}\right) \tag{8c, d}$$