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Bending Moment-Axial Force-Curvature Interactions for Metal Beam-Column Sections

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ABSTRACT

A simple numerical method capable of determining the *M-P-Φ* interaction diagrams of metal beam-columns sections subjected to axial load and bending moment (axial or biaxial) is presented. The effects of residual stresses, strain hardening and local buckling can be included. Tubular beam-columns and other shapes made of steel, aluminum or other metals can be analyzed by dividing the cross section in rectangular segments or in discrete areas. The method is based on the classic Euler-Bernoulli assumption that plane cross sections remain plane after deformations take place. The effects of shear stresses and strains are not taken into account. The maximum bending moment of the moment-curvature relationship for a given level of axial load is used to construct the failure surface for short beam-columns. The proposed model can be easily programmed by end users becoming an alternative to more refined and complex finite element models available in the technical literature and its validity and is verified against experimental results as well as those obtained using other analytical methods available in the technical literature.

1. Introduction

The behavior of metal beam-columns subjected simultaneously to axial load and biaxial bending moments are of great importance in structural engineering. In the case of steel members, their behavior has been investigated extensively by many structural researchers during the last seven decades. Niimoto [1] used the virtual displacement method to predict the ultimate strength of rectangular steel columns loaded eccentrically. Yu [2] investigated the buckling of concentric loaded columns without residual stresses. Chen and Santathadaporn [3] and Santathadaporn and Chen [4,5] used generalized stress-strain relationships in the stability analysis and failure of steel columns. Nishino and Tall [6] used finite differences to study the effects of residual stresses on the local buckling of columns. Atsuta [7] presented an analytical method for calculating the inelastic behavior of steel columns of different cross sections subjected to biaxial bending moments.

Fiala [8] presented a model for the bending moment-curvature-thrust curves of hybrid steel sections. Tebedge and Chen [9] presented a formulation for elastic-plastic failure curves of H-shaped columns. Szuladzinski [10] proposed equations for bending moment-curvature diagrams of circular tubes using nonlinear stress-strain relationships. Ross and Chen [11] presented a theoretical model for the moment-curvature diagrams of tubular sections under biaxial bending. Ross and Chen [12] proposed an expression for the failure surface of steel I-columns. Zhou and Chen [13] and Uchida and Morino [14] studied the effects of biaxial bending and residual stresses on the behavior and strength of box columns. Duan and Chen [15] presented equations for the design of both short and slender steel I-columns under biaxial bending. Shoal et al. [16] and Duan and Chen [17] presented *P-M-Φ* curves of tubular columns.

Landet and Lotsberg [18] presented experimental moment-curvature curves of dented tubular columns. Duan et al. [19] proposed closed-form expressions for the moment-curvature diagrams of dented tubular columns. Fan [20] and Bruin [21] developed models for the moment-curvature diagrams of dented tubular elements used in oil rigs. Charalampakis [22] proposed a formula for the interaction diagram of cross sections made of L-angles of equal legs subject to axial load and biaxial bending. More recently, Liew and Gardner [23] studied the behavior and failure surfaces of box- and I-shaped columns including the effects of strain hardening.

Research on the behavior of cross sections made of other metals also has increased in the last decades, although there are some aspects such as the behavior under biaxial bending that still requires further research. Sidebottom and Clark [24] studied the theoretical and experimental behavior of members with L- and T-shapes made of aluminum

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subject to eccentric axial load. Sully and Hancock [25] studied the behavior of stainless steel columns made of square hollow sections (SHS) subject to eccentric axial load. Reyes et al. [26] studied the behavior of aluminum square tubes subject to oblique loading. More recently, Bulson [27] presented a compilation of research carried out in Europe on aluminum structures.

The main objective of this article is to present a simple method for the calculation of *M-P-Φ* interaction diagrams for metal columns of arbitrary cross section subject to uni- or bi-axial bending. The maximum values of these diagrams can be used to construct the failure curves for short columns. In addition, the *M-P-Φ* interaction diagrams can be used in the calculation of rotations and deflections along the span of slender beam-columns by numerical integration. However, this application is currently under investigation by the authors of this publication.

2. Proposed analytical model

Consider the arbitrary tubular section of a metal column (Fig. 1) subject simultaneously to axial load and biaxial bending. The position of the neutral axis is defined by the intercept with the *Y*-axis (denoted as b) and the angle that it makes with the *X*-axis (denoted as α). The origin O of the global *XY*-system coincides with the centroid of the cross section. The *XY*-system is used to define both the geometry of the column cross section and the eccentricity of the applied axial load.

On other hand, the local *xy*-system is defined by the neutral axis and the normal line to the neutral axis containing the farthest point of the cross section in compression with coordinates (X_e, Y_e) with respect to the *XY*-system. This point is located at a distance c from the *x*-axis defining the depth of the neutral axis. The local *xy*-system is used in all numerical integrations. The origin O' of the local *xy*-system has coordinates X_a, Y_a with respect to the global *XY*-system. Using Fig. 1 the following geometric relationships can be obtained:

$$c = \frac{|\tan\alpha\, X_e - Y_e + b|}{\sqrt{\tan^2\alpha + 1}} \tag{1}$$

$$X_e = X_a + c\ \sin\alpha \tag{2}$$

$$Y_e = Y_a + c\ \cos\alpha \tag{3}$$

2.1. Resulting axial force and biaxial moments

Two models are used to calculate approximately the resulting axial load and biaxial moments for a given cross section of the member using: 1) discrete areas; and 2) polylines.

2.1.1. Approximation by discrete areas

In this model the cross section is divided into small areas a_i each with its centroid location defined by the coordinates X_i and Y_i as shown in Fig. 2. The axial strain at the centroid of area a_i for a given position of the neutral axis (i.e., b and α) is as follows

$$\varepsilon_i = \frac{y_i}{c}\varepsilon_c + \varepsilon_{ri} \tag{4}$$

where ε_c = maximum axial shortening strain in the cross section;
ε_{ri} = residual axial strain (positive for shortening and negative for elongation); and

$$y_i = (Y_i - Y_a)\ \cos\alpha + (X_i - X_a)\ \sin\alpha \tag{5}$$

The main advantage of the proposed model is its capacity to consider any distribution of residual stresses present in the cross section unlike the model of discrete polylines which does not have this capacity. The material strain-stress relationship is approximated using polylines as shown in Fig. 3 for both its ascending (loading) and descending (unloading) parts. Any material can be modeled including those with different behavior in tension from that in compression. Consequently, the stress σ_i at the centroid of area a_i can be calculated by linear interpolation as follows:

$$\sigma_i = \frac{\sigma_1}{\varepsilon_1}\varepsilon_i \quad \text{for}\ \varepsilon_i \ge \varepsilon_1\ \text{(in tension), and}\ \varepsilon_i \le \varepsilon_1\ \text{(incompression)} \tag{6}$$

$$\sigma_i = \frac{\sigma_j - \sigma_{j-1}}{\varepsilon_j - \varepsilon_{j-1}}\varepsilon_i + \sigma_{j-1} - \frac{\sigma_j - \sigma_{j-1}}{\varepsilon_j - \varepsilon_{j-1}}\varepsilon_{j-1} \tag{7}$$

Eq. (7) is also valid for both tension and compression stresses.

In the approximation by discrete areas, the properties of each discrete area are concentrated at its centroid. Now the accuracy problems associated with discrete areas located near the neutral axis can be reduced by refining the mesh at those locations.

2.1.2. Approximation by discrete polylines

In this approach the cross section is approximated using inter-connected linear segments as shown in Fig. 4. The area segment i is defined by the coordinates of its two ends using the global system *XY* and assuming constant thickness h_i along its length. The depth c of the neutral axis is defined by the line segment with the farthest end from the *x*-axis. The contributions to axial strength and biaxial internal moments of each segment used to approximate the shape of the section can be calculated as follows:

$$P_{1j} = \int_{y_{\min}}^{y_{\max}} \sigma dA \tag{8}$$

$$M_{1xj} = \int_{y_{\min}}^{y_{\max}} \sigma y dA \tag{9}$$

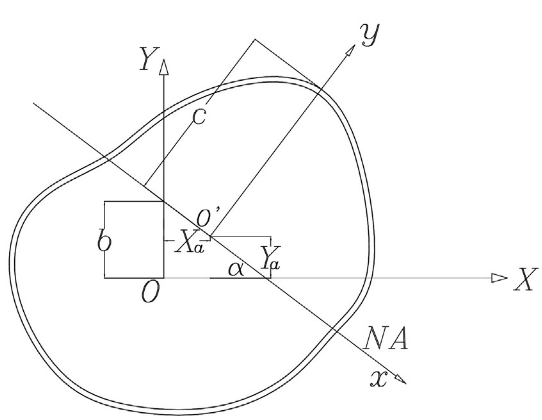

Fig. 1. Global and local systems of axes and arbitrary cross section of the member.

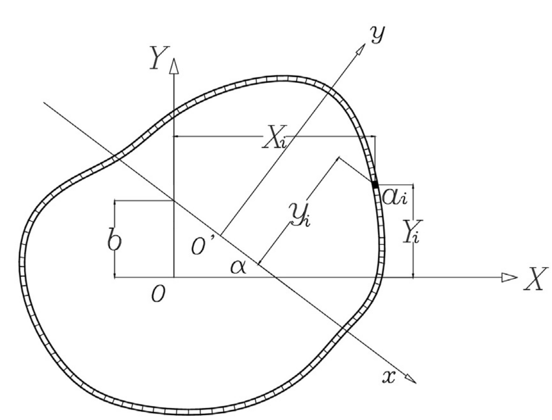

Fig. 2. Approximation by discrete areas.