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Koiter Asymptotic Analysis of Thin-walled Cold-formed Steel Uprights Pallet Racks

V. Ungureanu [a,b], A. Madeo [c,*], G. Zagari [c], G. Zucco [c], D. Dubina [a,b], R. Zinno [c]

[a] Politehnica University of Timisoara, Romania
[b] Romanian Academy, Timisoara Branch, Romania
[c] University of Calabria, Italy

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ABSTRACT

An imperfection sensitivity analysis of cold-formed steel pallet racks in compression is presented. The analysis is based on Koiter's approach and Monte Carlo simulation on one hand, and the ECBL (Erosion of Critical Buckling Load) approach on the other one. Mode interaction is taken into account and, based on that, the limit load and erosion of critical buckling load are evaluated. The analysis is based on an intensive experimental study carried out at the Politehnica University of Timisoara and extended to other thicknesses of the cross-section in order to highlight the variation of erosion. Thousands of imperfections were analyzed at a very low computational cost and an effective statistical evaluation of the limit performance was carried out.

1. Introduction

The finite element implementation of Koiter's asymptotic approach [1–3] allows the evaluation of pre-critical and initial post-critical behavior of slender elastic structures, also in the presence of strong nonlinear pre-critical behavior and in the case of interactive buckling. The method is considered very attractive for its advantages compared to the path-following approach [4]. These consist in an accurate post-buckling analysis and in an efficient imperfection sensitivity analysis with low computational cost [1]. The main difficulties arise in the availability of geometrically coherent structural model and in an accurate evaluation of their high order energy variations [5,6]. The use of corotational formulation, within a mixed formulation, allows to have a general finite element implementation of Koiter's analysis [7,8]. Our recent technology [9–11], in terms of numerical implementation, is applied to the evaluation of the performance of slender cold-formed steel members especially for the case of modal interaction. In particular, an efficient and robust imperfection sensitivity analysis is performed. Using a Monte Carlo simulation, for a random sequence of imperfections assumed as having the shape as linear combinations of buckling modes, the equilibrium paths for the imperfect structures are recovered. The load carrying capacity is evaluated statistically. The worst imperfections are detected and the limit load is obtained allowing the evaluation of the erosion of the critical bifurcation load using ECBL approach [12].

2. Koiter's asymptotic analysis

2.1. Fundamental equations

The finite element implementation of Koiter asymptotic analysis is essentially the implementation of Koiter's nonlinear elastic stability approach [13] into the finite element method (FEM) [1]. The solution process is based on an expansion of the potential energy in terms of load factor λ and modal amplitudes ξ_i. It can be summarized as follows:

1. The *fundamental path* is obtained as a linear extrapolation

$$\boldsymbol{u}^f[\lambda] = \boldsymbol{u}_0 + \lambda\hat{\boldsymbol{u}} \tag{1a}$$

where $\boldsymbol{u}_0$ is an initial displacement, possibly null, and $\boldsymbol{u} = \lambda\hat{\boldsymbol{u}}$ .is the vector of kinematic parameters, i.e., the degrees of freedom (dof) of the structure and $\hat{\boldsymbol{u}} = d\boldsymbol{u}/d\lambda$ is obtained as the solution of the linear algebraic equation

$$\boldsymbol{K}_0\hat{\boldsymbol{u}} = \hat{\boldsymbol{p}} \tag{1b}$$

where $\hat{p}$ is the reference load and $\boldsymbol{K}_0 = \boldsymbol{K}[\boldsymbol{u}_0]$ is the stiffness matrix, which contains the coefficients of the quadratic terms of the energy Φ''.

* Corresponding author at: University of Calabria, Arcavacata di Rende, CS, Italy.
E-mail address: antonio.madeo81@unical.it (A. Madeo).

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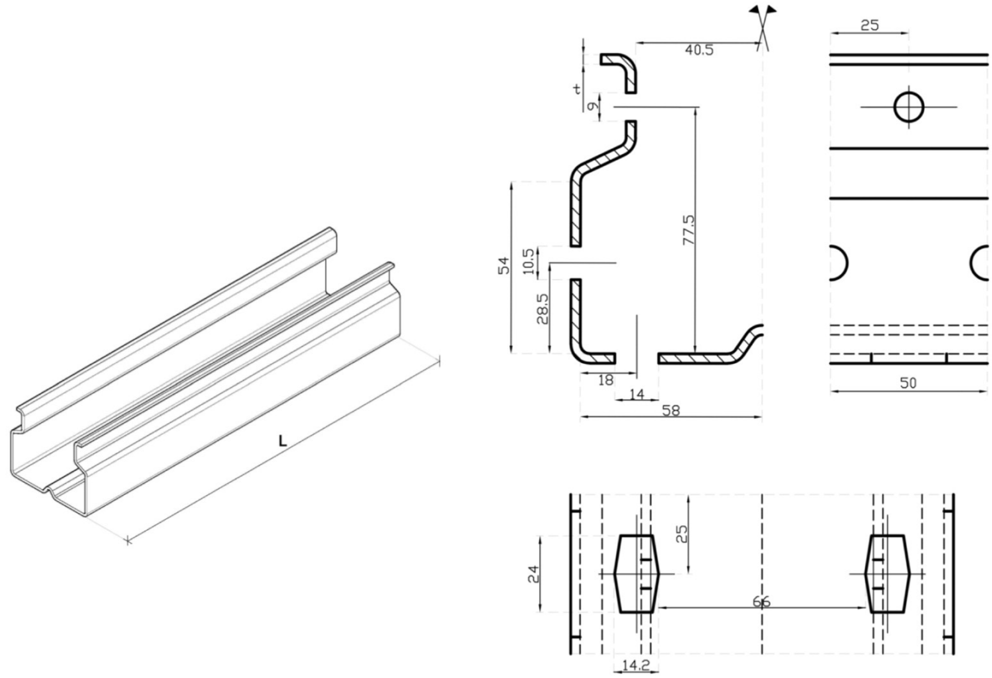

Fig. 1. Geometry of RS 125 × 3.2 with and without perforations (dimensions are expressed in mm).

2. A cluster of *buckling loads* λ_i, $i = 1 \cdots m$, and associated *buckling modes* $\dot{\boldsymbol{v}}_i$ are obtained along $\boldsymbol{u}^f[\lambda]$ by the critical condition

$$\boldsymbol{K}[\lambda_i]\dot{\boldsymbol{v}}_i = 0, \boldsymbol{K}[\lambda] = \boldsymbol{K}[\boldsymbol{u}_0 + \lambda\hat{\boldsymbol{u}}]. \quad (1c)$$

The eigenvalue problem is defined as fully non-linear to correctly recover the post-critical behavior. λ_b denotes an appropriate reference value for the cluster, e.g. the smallest of λ_i or their mean value, and with a suffix "b" the quantities evaluated corresponding to $\boldsymbol{u}_b = \boldsymbol{u}^f[\lambda_b]$. The buckling modes are normalized using the orthogonality condition

$$\Phi'''_b \hat{u}\dot{v}_i\dot{v}_j = 0 \quad , \quad \forall i, j = 1 \ldots m \quad (1d)$$

where Φ'''_b is the third order derivative evaluated at λ_b.
Note that the size m of the subspace of buckling modes needed for the analysis is of an order of magnitude smaller than the number of dof used to discretize the structure.
$\boldsymbol{V} = \{\dot{\boldsymbol{v}} = \sum_{i=1}^{m} \xi_i \dot{\boldsymbol{v}}_i\}$ denotes the subspace spanned by the buckling modes and $\dot{\boldsymbol{v}}_i$ and $\boldsymbol{W} = \{\boldsymbol{w} : \boldsymbol{w} \perp \dot{\boldsymbol{v}}_i, i = 1 \cdots m\}$ its orthogonal complement according to the orthogonality condition

$$\boldsymbol{w} \perp \dot{\boldsymbol{v}}_i \Leftrightarrow \phi_b''' \hat{u}\dot{v}_i w = 0 \quad (1e)$$

where $\hat{u} = \boldsymbol{\mathcal{L}}\ \hat{\boldsymbol{u}}, \dot{v}_i = \boldsymbol{\mathcal{L}}\dot{\boldsymbol{v}}_i, w = \boldsymbol{\mathcal{L}}\boldsymbol{w}$ and $\boldsymbol{\mathcal{L}}$ denotes the linear operator of FEM interpolation.

3. With $\xi_0 = (\lambda - \lambda_b)$ and $\dot{\boldsymbol{v}}_0 = \hat{\boldsymbol{u}}$, the asymptotic approximation for any equilibrium path is approximated by an expansion in terms of mode amplitudes ξ_j is as follows

$$\boldsymbol{u}[\lambda, \xi_k] = \boldsymbol{u}_b + \sum_{i=0}^{m} \xi_i \dot{\boldsymbol{v}}_i + \frac{1}{2}\sum_{i,j=0}^{m} \xi_i \xi_j \boldsymbol{w}_{ij} \quad (1f)$$

where $\boldsymbol{w}_{ij} \in \boldsymbol{W}$ are quadratic corrections introduced to satisfy the projection of the equilibrium equation into $\mathcal{W}$, obtained by the linear *orthogonal equations*

$$\delta \boldsymbol{w}^T \left(\boldsymbol{K}_b \boldsymbol{w}_{ij} + \boldsymbol{p}_{ij}\right) = 0, \quad \forall \boldsymbol{w} \in \mathcal{W} \quad (1g)$$

where $\boldsymbol{K}_b = \boldsymbol{K}_0 + \lambda_b \boldsymbol{K}_1$ and vectors $\boldsymbol{p}_{ij}$ are defined as a function of modes $\dot{\boldsymbol{v}}_i$ and, $i = 0 \cdots m$ by the energy equivalence $\delta \boldsymbol{w}^T \boldsymbol{p}_{ij} = \Phi_b''' \delta w \dot{v}_j \dot{v}_j$.

4. The following energy terms are computed for $i,j = 0 \cdots m$, $k = 1 \cdots m$

$$\begin{aligned} A_{ijk} &= \phi_b''' \dot{v}_i \dot{v}_j \dot{v}_k \\ B_{ijhk} &= \phi_b'''' \dot{v}_i \dot{v}_j \dot{v}_h \dot{v}_k - \phi_b''(w_{ij}w_{hk} + w_{ih}w_{jk} + w_{ik}w_{jh}) \\ C_{ik} &= \phi_b'' w_{00} w_{ik} \\ \mu_k[\lambda] &= \frac{1}{2}\lambda_b\left(\lambda - \frac{1}{2}\lambda_b\right)\phi_b''' \hat{u}^2 \dot{v}_k + \frac{1}{6}\lambda_b^2(\lambda_b - 3\lambda)\phi_b'''' \hat{u}^3 \dot{v}_k \end{aligned} \quad (1h)$$

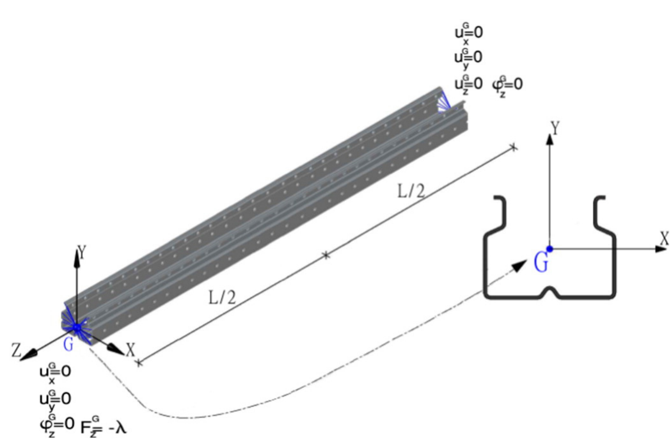

Fig. 2. Geometry and boundary conditions of the member.

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