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Optimal design of two-dimensional porosity distribution in shear deformable functionally graded porous beams for stability analysis

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ABSTRACT

In the present study, considering two-dimensional porosity distribution through a functionally graded porous (FGP) beam, its optimal distributions are obtained. A multi-objective optimization problem is defined to maximize critical buckling load and minimize mass of the beam, simultaneously. To this end, Timoshenko beam theory is employed and equilibrium equations for two-dimensional functionally graded porous (2D-FGP) beam are derived. For the solution, we present generalized differential quadrature method (GDQM) and consider two symmetric boundary conditions (Clamped-Clamped and Hinged-Hinged). Solving generalized eigenvalue problem, critical buckling load for 2D-FGP beam is then obtained. During optimization procedure, a cubic polynomial spline interpolating on a finite number of design variables is considered as porosity distribution function. Solving the multi-objective optimization problem using bio-inspired genetic algorithm (NSGA II), leads to a set of optimal porosity distributions known as Pareto optimal solutions. To show the validity of the proposed formulation, we compare results with those reported (1D and 2D porosity distributions) in the literature as well as finite element simulations. We also compare Pareto solutions with optimization result of one dimensional porosity distribution which clearly demonstrates the importance of the presented optimization procedure. In general, optimum porosity distributions are different in each boundary condition. However, in most of the optimum solutions, middle line of the beam is composed of the material with higher values of porosity and outer corners have lower values of porosity. Pareto optimal solutions also indicate that, sharp decreasing of the mass makes a slight decline in critical buckling load when it has large values. The proposed approach can be used for design of porosity distribution in FGP structures.

1. Introduction

Porous materials have been widely used in many engineering applications, such as those related to lightweight structures, biomedical instruments and aerospace [1–3]. Inspired by natural bone, a paradigm shift is currently occurring in porous materials design from homogeneous porosity distribution to functionally graded one (heterogeneous porosity distributions) [4,5]. Tailoring the desired properties is then achievable by graded porosity distribution. Recently, functionally graded porous (FGP) structures with complex geometries have been fabricated using advanced manufacturing techniques, known as solid free-form fabrication or rapid prototyping [6]. Considering the advantages of FGP structures and improved fabrication technologies, naturally induces the investigation of porosity distribution.

The analysis of structures with uniform porosity distribution has been considered in several works. Simone and Gibson [7] demonstrated the suitability of uniformly distributed porosity structures in components, including honeycomb beams, sandwich panels, and cylindrical shells with porous cores. Wieding et al. [8] carried out a finite element analysis on the biomechanical stability of cylindrical porous titanium to act as bone scaffolds. Magnucki et al. [9] considered bending and buckling analysis of a rectangular porous plate to demonstrate the influence of porosity and thickness on the critical buckling load. Jabbari et al. [10] proposed buckling analysis of a circular plate with uniform porosity distribution, investigating the effects of different porosity and thickness values.

Stability and vibration analysis of FGP structures with a prescribed one-dimensional (1D) porosity distributions have been studied in some works. Chen et al. [11,12] conducted stability, bending, free and forced vibration analysis of FGP beams while porosity varies along thickness. The results demonstrate fine performance of symmetric porosity distribution. In another study, similar 1D porosity distribution has been considered in order to analyze nonlinear free vibration of a sandwich beam with a FGP core [13]. Ebrahimi and Zia [3] investigated nonlinear vibration characteristics of functionally graded (FG) Timoshenko beams made of porous materials. Feyzi and Khorshidvand [14] carried

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out an analytical study of a FGP circular plate post-buckling saturated by fluid, where porosity varies through the thickness. The results demonstrate a noticeable effect of porosity distribution function on the results. Mojahedin et al. [15] similarly conducted buckling analysis of FGP circular plates based on higher order shear deformation theory. Hadji and Adda Bedia [16] implemented a new displacement field based on refined shear deformation theory to present free vibration of FGP beam with porosity distribution along thickness direction. Zhang et al. [4] discussed the production procedure of FGP shape memory alloys and demonstrated their good performance in comparison with uniformly distributed porosity structures. Moreover, Yahia et al. [17] considered FG plates with porosity phases occurred during fabrication process. Results are useful especially for the ultrasonic inspection techniques and structural health monitoring.

It is quite clear that optimization of porosity distribution can be an eminently effective method to improve structural characteristics, such as mass and critical buckling load. Despite the importance of this topic, there are just limited works related to volume fraction optimization in the FG structures, which can somehow be considered similar to the optimization of porosity distribution. Asgari [18] dealt with the optimization of volume fraction in a thick hollow FG cylinder in order to improve wave propagation behavior. In another study, the same author implemented this approach to find optimal material distribution for a prescribed temperature field in transient heat conduction [19]. The results indicate that employing the optimal material distribution can considerably affect objective functions, such as distribution of temperature and mass, compared to the power law volume fraction distribution. Goupee and Vel [20] have also proposed optimal material distributions of FG materials to simultaneously minimize the mass and maximize the factor of safety for steady thermomechanical processes. Wieding et al. [21] considered optimization of open-cell porous structures to match the elastic properties of human cortical bone. They demonstrated that the optimization procedure can be a beneficial tool to reduce the amount of the required material without affecting the biomechanical performance of the structure.

Since stability analysis and optimal tailoring of material distribution have great significance, in the present study despite the conventional 1D porosity distribution, the optimization of porosity distribution in a 2D-FGP beam is considered. To this end, we study a FGP beam where porosity varies through the thickness and longitudinal directions. The objective functions are mass and critical buckling load, which are simultaneously minimized and maximized, respectively. In Section 2, the virtual work principle is employed to derive equilibrium equations of a 2D-FGP Timoshenko beam. We also present the generalized differential quadrature method (GDQM) for the solution of derived equations. The critical buckling load for 2D-FGP beams of unknown porosity distributions under two symmetric boundary conditions (BCs), i.e., Hinged-Hinged and Clamped-Clamped are also presented. In Section 3, a multi-objective optimization problem is proposed to simultaneously maximize critical buckling load and minimize mass where porosity distribution is considered to be unknown in the optimization problem. The porosity values on a finite number of points in the beam are then considered as design variables. We propose a cubic polynomial spline in order to determine porosity value at any arbitrary point. The design variables are then optimally determined employing the non-dominated sorting genetic algorithm (NSGA II). In Section 4, we validate the results of the presented formulation with a 1D porosity distribution available in the literature and 2D porosity distribution using finite element simulation in commercial software (ABAQUS). Optimal solutions, known as Pareto optimal solutions, are also compared with the optimization results of 1D porosity distribution under different BCs. Finally, we draw some conclusions in Section 5.

2. Functionally graded porous beam

We assume a FGP beam with a rectangular cross-section of unit width, thickness h and length l, as shown in Fig. 1. Porosity and consequently Young's modulus vary both in the thickness and longitudinal directions, and are related as [22]:

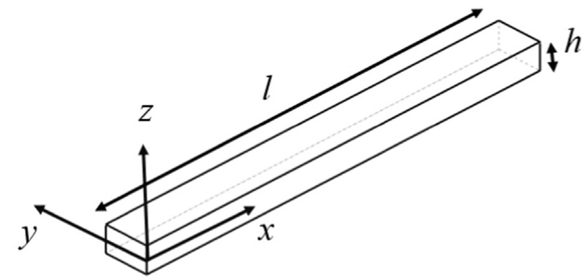

Fig. 1. Schematic of functionally graded porous beam.

$$\frac{E(x,z)}{E_{\max}} = (1 - f(x,z))^2 \tag{1}$$

where $E(x,z)$ and $f(x,z)$ demonstrate, respectively, the Young's modulus and porosity at an arbitrary point while $E_{\max}$ denotes the Young's modulus of the dense material. It should be noted that the Poisson's ratio ϑ is considered constant, as already assumed in several studies [11,12,22].

For the analysis of the FGP beam, the Timoshenko beam theory is employed, in which the displacement field can be presented as:

$$u(x,z,t) = u_0(x,t) + z\theta(x,t) \tag{2}$$

$$w(x,z,t) = w_0(x,t) \tag{3}$$

where u and w are displacements along x and z directions at time t in terms of the mid-surface ($z = 0$) displacements (u_0 and w_0) and rotation of the cross-section (θ).

The non-zero components of linear strain in terms of mid-surface displacements and rotation are then expressed as:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} + z\frac{\partial \theta}{\partial x} \tag{4}$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = \frac{\partial w_0}{\partial x} + \theta \tag{5}$$

where ε_{xx} and γ_{xz} are normal and transverse shear strains, respectively.

According to the generalized Hook's law, normal stress σ_{xx} and shear stress σ_{xz} can be written as:

$$\sigma_{xx} = Q_{11}(x,z)\,\varepsilon_{xx}, \qquad \sigma_{xz} = Q_{55}(x,z)\gamma_{xz} \tag{6}$$

where:

$$Q_{11}(x,z) = \frac{E(x,z)}{(1-\vartheta^2)}, \quad Q_{55}(x,z) = \frac{E(x,z)}{2(1+\vartheta)} \tag{7}$$

The relationship between stress resultants and stress components can also be expressed as [23,24]:

$$N_x = \int_{-h/2}^{h/2} \sigma_{xx} dz \tag{8}$$

$$M_x = \int_{-h/2}^{h/2} z\sigma_{xx} dz \tag{9}$$

$$Q_x = \int_{-h/2}^{h/2} k\sigma_{xz} dz \tag{10}$$

where is the shear correction factor considered normally for rectangular section [24,25]. Substituting Eqs. (4), (5) and (7) into Eq. (6), and rewriting Eqs. (8)–(10), the stress resultants are obtained as:

$$N_x = A_{11}(x)\frac{\partial u_0}{\partial x} + B_{11}(x)\frac{\partial \theta}{\partial x} \tag{11}$$

$$M_x = B_{11}(x)\frac{\partial u_0}{\partial x} + D_{11}(x)\frac{\partial \theta}{\partial x} \tag{12}$$