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Instability of a delaminated composite beam subjected to a concentrated follower force

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ABSTRACT

Vibration, buckling and flutter instability of delaminated composite beams are studied in this paper. An analytical solution is presented for instability of a composite beam with a single delamination subjected to concentrated follower force. Using Euler-Bernoulli beam theory and Classical Lamination Theory, equation of motion of a delaminated beam is derived and solved analytically for free vibration, buckling and flutter instability. After validating the results, effect of delamination location, delamination length and stacking sequence on fundamental frequency, buckling load and flutter load are studied. Results show that for a cantilever beam, when delamination moves from surface of the beam to its mid-plane, fundamental frequency and buckling load decreased while flutter load is increased.

1. Introduction

Due to numerous advantages of fiber-reinforced laminated composites such as high stiffness to density ratio and best match the design requirements of a specific structural application, these materials are used extensively in many industries. For this reason, many researchers are interested in studying the behavior of composite structures such as stress analysis [1,2], free vibration [3,4], linear and non-linear buckling [5,6] and dynamic stability [7–9]. Nevertheless, the analysis of these materials is far more complex when compared to conventional materials due to different types of coupling produced in these anisotropic materials.

Laminated composites are subjected to delamination because of high inter-laminar stresses, impact damage and fabrication defects. When a structure member delaminates, stresses within the member will be redistributed which affect the response of the structural. Therefore, it is necessary to understand and consider these changes in composite structural design. Yang and Oyadiji [10] presented a solution for detecting delamination in composite structures, using frequency deviations. They studied effects of delamination due to concentrated mass loading with respect to modal frequency variations in composite beams, and consequently employed frequency curves as NDT tool for delamination identification and localization. Kharghani and Soares [11] studied behavior of laminated composite plates with embedded delamination using a Layerwise Higher Order Shear Deformation Theory. They also predicted the initiation of debonding in opening and sliding fracture modes. Marjanovic et al. [12] used a simple and efficient algorithm to track a moving delamination front of arbitrary shape, using a laminated finite plate element model in conjunction with the Virtual Crack Closure Technique (VCCT). Shokrieh et al. [13] presented a modified model for simulation of mode I delamination growth in laminated composite materials. Aslan and Daricik [14] investigated the effect of multiple delaminations on the compressive, tensile and flexural strength of E-glass/epoxy composites and evaluated their effects on the first critical buckling load.

Due to extensive use of composites in aerospace structures, vibration, buckling and flutter instability of these materials are very important in design of structures. Jafari-Talookolaei et al. [15] presented analytical and finite element solutions for free vibration analysis of delaminated composite curved beams by taking into account the effects of shear deformation, rotary inertia, deepness terms and material coupling. Della [16] developed an analytical solution to study the free vibration of composite beams with two overlapping delamination under axial compressive load. His results show a linear relation between the square of the constrained mode and free mode frequencies of the simply supported beam with the axial compressive load. Park et al. [17] studied the free vibration of laminated composite skew plates with delamination based on the high-order shear deformation theory. Liu and Shu [18] developed an analytical solution to study free vibration of rotating Timoshenko beams with multiple delaminations. They also studied the influences of Timoshenko effect and rotating speed on vibration of delaminated beams. Kharazi et al. [19] presented an analytical method to study the buckling behavior of the composite plates with through-the-width delaminations by different plate theories.

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Ovesy et al. [20] studied the effect of through-the-width delamination on the dynamic buckling behavior of composite plates using semi-analytical finite strip method. Sahoo et al. [21] investigated the free vibration, bending and transient responses of delaminated composite plates using higher-order shear deformation theory in conjunction with finite element steps. They obtained final form of the governing equations of the bending and free vibration response using variational method and the classical Hamilton's principle. Tafreshi [22] presented a finite element method to analyze instability of delaminated composite cylindrical shells subjected to pure bending and also combined bending and axial compression. Rezaeepazhand and Wisnom [23] investigated the applicability of scaled models for predicting the buckling behavior of delaminated composite beams and studied the limitations and acceptable intervals of all parameters and corresponding scale factors.

Wang et al. [24] studied stability of an isotropic delaminated beam subjected to a follower force. They also investigated the effect of the location and length of the delamination on the buckling and flutter instability of the beam. Yazdi and Rezaeepazhand [25,26] used similitude theory and investigated flutter pressure of a delaminated composite beam-plate subjected to supersonic flow. Moreover, Yazdi [27] investigated flutter instability of delaminated cross-ply composite shells and panels when subjected to supersonic flow parallel to its length edge.

Dynamic stability analysis of delaminated composite structures is very complex and there is no exact solution for many of these types of problems, therefore researchers usually use numerical methods to solve these problems. In this paper, a simple analytical solution is presented to find flutter load of a laminated composite beam with a single delamination when subjected to a concentrated follower force. The length to thickness ratio of the beam is high and so Euler-Bernoulli beam theory can be used. Furthermore, all deflections are small and CLT is used for composite analysis. First, using Euler-Bernoulli beam theory and CLT, stability equation of the beam is derived and solved analytically for free vibration, buckling and flutter instability. Then results for free vibration and buckling are compared with previous investigations. Moreover, the effect of delamination length and location and stacking sequences of laminates on the flutter load of the laminated beam is investigated.

2. Governing equations

Fig. 1 shows one-dimensional model of a laminated composite beam with a delamination of length a. Length, thickness and width of the beam are L, h and b respectively. Beam is subjected to a concentrated follower load, P. The delamination divides the beam into four subbeams with length l_i, $i = 1 - 4$ as it is shown in Fig. 1. h_i, $i = 1 - 4$ are thicknesses of subbeams, which for subbeams 1 and 4 are equal to h. Each subbeam is subjected to load P_i, $i = 1 - 4$.

Let $W_i(x_i, t)$ be the small deflection for each subbeams at any point of them, f_i the deflection of each subbeams at its end and φ_i the angle of rotation of each subbeam's end section. Within the framework of usual assumptions of the elementary theory of bending, the equation of small deflection of each subbeams is of the form

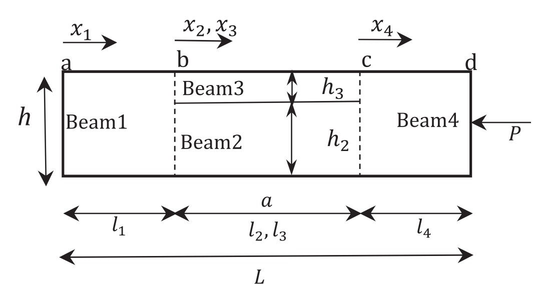

Fig. 1. Laminated composite beam with a single delamination.

$$-M_{xx}^i = P_i(f_i - W_i) - P_i\varphi_i(l_i - x_i) + L_j^i,\ i = 1 - 4 \tag{1}$$

In Eq. (1) M_{xx}^i is bending moment of each subbeams in x direction and L_j^i is the bending moment produced by the action of inertia forces (in the sense of d'Alembert principle) of each subbeams. Differentiating Eq. (1) twice and noting that

$$\frac{\partial^2 L_j^i}{\partial {x_i}^2} = -\rho_i \frac{\partial^2 W_i}{\partial t^2} \tag{2}$$

Eq. (3) is obtained where ρ_i is the mass per unit length of each subbeams.

$$-\frac{\partial^2 M_{xx}^i}{\partial {x_i}^2} + P_i\frac{\partial^2 W_i}{\partial {x_i}^2} + \rho_i\frac{\partial^2 W_i}{\partial t^2} = 0,\ i = 1 - 4 \tag{3}$$

According to CLT in-plane forces and moments for a laminated composite beam are in the form of Eqs. (4) and (5).

$$\widetilde{N} = \boldsymbol{A}\widetilde{\varepsilon}^0 + \boldsymbol{B}\widetilde{\kappa}^0 \tag{4}$$

$$\widetilde{M} = \boldsymbol{B}\widetilde{\varepsilon}^0 + \boldsymbol{D}\widetilde{\kappa}^0 \tag{5}$$

where N_i and M_i are in-plane forces and moments respectively, ${\varepsilon_i}^0$ are the mid-plane strains and ${\kappa_i}^0$ are mid-plane curvatures. A, B and D are extensional, extensional-bending coupling and bending stiffness matrices respectively, which calculated in unit length. Since P_i is the only axial load in each subbeams, it can be assumed that N_{xx}^i is the only non-zero in-plane force for each subbeams. Furthermore, because length to thickness and length to width ratios of the beam are high, M_{xy}^i and M_{yy}^i are very small, and so negligible. Also it is assumed that there is no contact between the delamination surfaces so they deform freely without any touching and have different transverse deformation. Therefore, the coupling only occurs at the crack tips. Using these assumptions and mid-plane strains and curvatures definitions, similar to [28] Eqs. (6)–(9) are obtained for each subbeams.

$$N_{xx}^i = b_i(A_{11}^i\frac{\partial U_i}{\partial x_i} + A_{12}^i\frac{\partial V_i}{\partial y_i} + A_{16}^i\left(\frac{\partial U_i}{\partial y_i} + \frac{\partial V_i}{\partial x_i}\right) - B_{11}^i\frac{\partial^2 W_i}{\partial {x_i}^2}) \tag{6}$$

$$N_{yy}^i = b_i(A_{12}^i\frac{\partial U_i}{\partial x_i} + A_{22}^i\frac{\partial V_i}{\partial y_i} + A_{26}^i\left(\frac{\partial U_i}{\partial y_i} + \frac{\partial V_i}{\partial x_i}\right) - B_{12}^i\frac{\partial^2 W_i}{\partial {x_i}^2}) = 0 \tag{7}$$

$$N_{xy}^i = b_i(A_{16}^i\frac{\partial U_i}{\partial x_i} + A_{26}^i\frac{\partial V_i}{\partial y_i} + A_{66}^i\left(\frac{\partial U_i}{\partial y_i} + \frac{\partial V_i}{\partial x_i}\right) - B_{16}^i\frac{\partial^2 W_i}{\partial {x_i}^2}) = 0 \tag{8}$$

$$M_{xx}^i = b_i\left(B_{11}^i\frac{\partial U_i}{\partial x_i} + B_{12}^i\frac{\partial V_i}{\partial y_i} + B_{16}^i\left(\frac{\partial U_i}{\partial y_i} + \frac{\partial V_i}{\partial x_i}\right) - D_{11}^i\frac{\partial^2 W_i}{\partial {x_i}^2}\right),\ i = 1 - 4 \tag{9}$$

Using Eqs. (7) and (8), Eqs. (6) and (9) can be rewritten only in terms of U_i and W_i and simplified in the form of Eqs. (10) and (11).

$$N_{xx}^i = b_i(\alpha_i\frac{\partial U_i}{\partial x_i} + \beta_i\frac{\partial^2 W_i}{\partial {x_i}^2}) \tag{10}$$

$$M_{xx}^i = b_i(\gamma_i\frac{\partial U_i}{\partial x_i} + \delta_i\frac{\partial^2 W_i}{\partial {x_i}^2}) \tag{11}$$

where

$$\alpha_i = A_{11}^i + \frac{1}{\Delta}(A_{26}^i {A_{16}^i}^2 - A_{16}^i A_{12}^i A_{66}^i + A_{26}^i {A_{12}^i}^2 - A_{12}^i A_{16}^i A_{22}^i) \tag{12}$$

$$\beta_i = -B_{11}^i + \frac{1}{\Delta}(A_{16}^i A_{66}^i B_{12}^i - A_{16}^i A_{26}^i B_{16}^i + A_{12}^i A_{22}^i B_{16}^i - A_{12}^i A_{26}^i B_{12}^i) \tag{13}$$

$$\gamma_i = B_{11}^i + \frac{1}{\Delta}(A_{16}^i A_{26}^i B_{12}^i - A_{12}^i A_{66}^i B_{16}^i + A_{12}^i A_{26}^i B_{12}^i - A_{16}^i A_{22}^i B_{12}^i) \tag{14}$$

$$\delta_i = -D_{11}^i + \frac{1}{\Delta}(B_{16}^i A_{66}^i B_{12}^i - A_{26}^i {B_{16}^i}^2 + B_{16}^i A_{22}^i B_{12}^i - A_{26}^i {B_{12}^i}^2)$$
$$\Delta = A_{22}A_{66} - {A_{26}}^2,\ i = 1 - 4 \tag{15}$$